TRADE MATHEMATICS:
A Handbook for Teachers
The TAFE National Centre for Research and Development was set up as a company in late 1981 by the State, Territory and Commonwealth Ministers for Education, to undertake TAFE research and development projects of national significance.
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Introduction

In 1983 and 1984 the NSW Department of TAFE's Staff Development Division presented a series of Trade Mathematics Workshops for teachers from several trade areas: Carpentry and Joinery, Plumbing, Automotive Engineering and Fitting and Machining. The emphasis was on teaching mathematics in normal lessons rather than on remediation techniques.

Separate workshops were developed for each trade. Trade teachers had preliminary meetings with the course presenters to discuss the areas which they thought were most difficult for students. The topics raised in these discussions were remarkably similar, so that common themes emerged: decimals, percentages, estimation, teaching concepts, formulas, written calculations and solving problems. These were combined into four workshop sessions with some changes from one trade to another. Matching notes were written as handouts for the teachers attending the workshops, each set being adapted using specific trade examples taken from student assignments, examination papers and textbooks. An underlying assumption was that students would be using calculators for most of their trade calculations.

This handbook is a sample package of the materials prepared for the Trade Mathematics Workshops. It has been put together by combining the notes used for the four trade areas and therefore contains a mixture of trade applications. Some chapters have worksheets which teachers will be able to use for class exercises, homework assignments or tests, without breach of copyright.

The principles emphasised in the book are teaching for understanding of concepts, keeping calculations as straightforward as possible and applying problem solving techniques in all contexts. We believe that not only trade teachers, but all teachers involved in vocational mathematics will find the material in this handbook stimulating, relevant and useful.

We would like to thank Sera Gandolfo for her careful preparation of the manuscript, and Penny Wilson for drawing the original diagrams.
Chapter 1: Decimals

Decimals have taken over in importance from fractions for recording parts of a whole. This change results from decimalisation of money, metrication of measurements and the use of calculators and computers. Most trade calculations are done with a calculator and these notes are written from this point of view. However, many teachers still expect students to be able to perform simple calculations with pen and paper, so some relevant teaching methods and worksheets are included in section 6. Even if students always use a calculator, they must understand the meaning of decimal notation, different ways of using zeros, order of size in decimals and rounding to a given number of decimal places, and these are discussed in sections 1 to 5.

1 Meaning of decimal notation

Students need a strong mental link between the number of decimal places and the number of equal parts, preferably without using fraction notation. Real life examples will help them to understand.

a. One decimal place is used to show 10 parts: petrol prices, timber lengths, trip meter distances.
b. Two decimal places are used to show 100 parts: supermarket-shelf price tags, race record times, e.g. 59.48s or 1:07.38.
c. Three decimal places are used to show 1000 parts: pre-packaged meats in supermarkets.

Ways of teaching the concept of decimals and the meaning of decimal notation include:

a. Discussion of recording instruments such as trip meters, cash registers and electronic scales, preferably with photographs or models.
b. Concrete aids, e.g. Dienes blocks, coins.
c. Comparison of the decimal system with other base 10 numbering: room numbers in high-rise buildings, car number plates, phone numbers.
d. Comparison of decimals to other systems not based on 10, e.g. parts of an over in cricket (5.2 = 5 overs 2 balls), ages of children (1.3 = 1y 3mo).
2 Use of zero in decimals

Zero is used in a variety of ways in decimal work.

a. Zero as a place holder, e.g. $16.08.

b. Zero as a type of punctuation, e.g. 0.75.

c. Zero to show degree of accuracy, e.g. $1.798 \approx 1.80$ (correct to 2 decimal places).

d. Zero in equivalent decimals, e.g. 1.6 on a calculator means 1.60 if you are working with money.

e. Filling in zeros:
   When adding or subtracting decimals with ‘ragged ends’, students should always fill in the missing zeros.

   Example 1: $0.36 + 1.9$ becomes $0.36 + 1.90$

   Example 2: $2.5 - 0.36$ becomes $2.50 - 0.36$

f. Zeros in metric measuring:
   It is helpful when recording lengths in metric to fill in three decimal places using zeros, e.g. 1.8m lengths of steel may be recorded as 1.800m, which gives a quick conversion to 1800mm.

g. Zeros in addition:
   A common error is $0.5 + 0.9$ using zeros helps to prevent this error $0.5 + 0.9$
   
   Using zeros helps to prevent this error $1.4$

3 Essential zeros

Essential zeros and non-essential zeros can be sorted out by a see-saw method. When a number is tilted a zero will roll off if it is not an essential zero*.

Note: A decimal point or another digit will stop zeros rolling off.

The idea of rolling zeros works for whole numbers also if you stress that all whole numbers have an invisible decimal point straight after the units digit.

4 Order of size in decimals

A way to test whether or not students have understood the concept of decimals and the use of zeros is to ask them to place several decimals in order from smallest to largest. This is a common criterion reference test item.

Example: Arrange from smallest to largest:
0.9, 0.096, 0.906, 0.96

Decimal Order Cards are a good teaching aid for order of size in decimals. If students cannot order the cards correctly it shows that some decimal concepts still need development.

A set of 24 cards of playing-card size are printed on one side with one- two- and three-digit decimals, 0.008 to 0.96. Students may work alone or in small groups. The cards are shuffled and drawn one at a time from the pack and placed in a row from smallest to largest. Without the teacher being involved the pack is played out and, when the cards are turned over, the correctness of the order chosen is revealed by small numbers, 1 to 24. If there are errors they have to be corrected with a minimum of teacher intervention. Some people put the smallest on the right, contrary to the conventions for the number-line and measuring, and this has to be altered. Self-correction brings out many misconceptions on decimal notation. Most students eventually decide on a method of mentally filling in zeros to three places; some use a system that is similar to alphabetical order.

Decimal Order Cards are available from the Division of Basic Education, NSW Department of TAFE.

5 Rounding

Some students ask, ‘Do decimals go on past three places?’

Students will come across decimals with four or more places in a variety of calculations: multiplying, dividing, squaring and taking square roots. They must get used to seeing strings of digits on their calculators. They will ask, ‘What do I do with all these numbers?’ Handling this requires rounding (or approximation).

Rounding to 1, 2 or 3 decimal places is an essential skill. Many students do not realise that answers in trade mathematics are often not exact; answers are as accurate as you want them to be for the practical problem you are solving.
The strategy for rounding

Look at the part of the number that is not needed. If there is half the unit of measure or more, take the answer up. If there is less than half, take the answer down.

Do some calculations with a non-decimal system such as time.

Examples: Write to the nearest hour, 4 hrs 38 mins. Write to the nearest week, 3 weeks 2 days.

When students understand the concept, change to decimals. The decimal system has 10 digits, 5 small and 5 large.

Small: 0 1 2 3 4 Large: 5 6 7 8 9
Small digits, make no change Large digits, go up one.

When rounding, give students something to do with their pen besides just sitting there and looking at the numbers! Draw a cut-off mark after the number of decimal places needed.

Example: Write 2.786 correct to 2 decimal places.
The cut-off mark goes after the 8: 2.78 | 6
Check the digit on the right of the cut-off mark to see if it is large or small. If large, go up one.
+ 1
2.78 | 6 Answer = 2.79

Use a variety of examples and include complicated ones with zeros. These will again remind students that 1.8 = 1.80 = 1.800, except when they show the degree of accuracy.

Examples: 1.796 to 2 decimal places = 1.80
4.996 to 2 decimal places = 5.00
2.03 to 1 decimal place = 2.0
7.999 to 1 decimal place = 8.0

6 Operating with decimals

Multiplying and dividing by 10, 100 and 1000.

To do this we teach students to move the point. However, there is a major source of confusion here. Is the decimal point moving, or are the numbers moving?

Example: Multiply 7.25 by 10, 100, and 1000.
10 × 7.25 = 72.5
100 × 7.25 = 725
1000 × 7.25 = 7250

Obviously, the 7, 2 and 5 are moving, not the point. However it is still better to say, 'Move the decimal point'. Reasons for sticking with this old rule are:

(i) because students have always done it this way at school.
(ii) because it is neat and tidy.
(iii) because we don't usually work with ruled columns.
Decimals

Bookmakers and sales assistants work in ruled columns and they move the numbers, not the point, to avoid mistakes.

*Example:* Find the cost of a book @ $16.80 if 10% discount is allowed.

<table>
<thead>
<tr>
<th>Cost</th>
<th>$16</th>
<th>80</th>
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<tbody>
<tr>
<td>less 10%</td>
<td>1</td>
<td>68</td>
</tr>
<tr>
<td>Balance</td>
<td>$15</td>
<td>12</td>
</tr>
</tbody>
</table>

Students who have trouble with keeping decimals under each other should turn their paper so that the lines become columns.

**Calculations where the point does not move**

Most calculations involving decimals do not cause the point to move. Practice in these questions gives students confidence before attempting harder examples.

See Worksheet 1.1: Calculations with Decimals, for a specimen set. As with most of the worksheets, answers at the bottom may be folded under while the questions are being worked.

**Multiplications and divisions where the decimal point does move**

Some teaching tips are helpful if students have to learn these skills without using a calculator because of syllabus requirements.

**Multiplication**

Count the decimal places in each part of the question and write these as little numbers on the side.

Add the little numbers and write their total on the far side in the answer line.

Multiply to get the answer, as if they were whole numbers. Place the point so that you have three decimal places.

<table>
<thead>
<tr>
<th>2.7 ( \times ) 0.38</th>
<th>2</th>
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</thead>
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<tr>
<td>2.7</td>
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<td>( \times ) 0.38</td>
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<td>21.6</td>
<td>3</td>
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<tr>
<td>81.0</td>
<td>3</td>
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<tr>
<td>102.6</td>
<td>3</td>
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</tbody>
</table>

**Division**

The hard part which worries students is knowing where the point goes in the answer. Use a low stress technique. After the decimal points have been moved to the correct positions, put a point up in the answer line before starting to divide.

*Example:* \( 2.95 \div 0.5 \)

**Step 1:**

\[
\begin{array}{c}
\underline{0.5 \overline{2.95}} \\
5 \underline{29.5}
\end{array}
\]

Multiply both numbers by 10 to change 0.5 to 5 and 2.95 to 29.5

\[
\begin{array}{c}
\underline{5 \overline{29.5}}
\end{array}
\]

Step 2: Place a very obvious decimal point in the answer line above the point in 29.5

\[
\begin{array}{c}
5 \overline{)29.5} \\
5 \cdot 5 \overline{)29.5} \\
\end{array}
\]

This point is placed before completing the calculation.

Step 3: Complete the calculation in the usual way.

\[
\begin{array}{c}
5 \overline{)29.5} \text{ (Correct)} \\
5 \cdot 5 \overline{)29.5} \\
\end{array}
\]

For exercises in multiplication and division of decimals where the point has to move see Worksheet 1.2: Harder Calculations with Decimals.

Worksheets of graded examples

Four more worksheets at the end of this chapter contain specimen sets of questions, graded from easy to hard, with each item slightly different or more complex than the one before. The worksheets are:

1.3 Addition with Decimals
1.4 Subtraction with Decimals
1.5 Multiplication with Decimals
1.6 Division with Decimals

All types of calculating are included except long division which is more likely to be done with a calculator. On Worksheet 1.5, Multiplication with Decimals, digits are supplied for the answers and only the decimal points have to be inserted. Worksheet 1.6 presents division in three different ways which are likely to be met in trade calculations.

Multiplication and division by numbers smaller than one

Students' concepts of multiplication and division were first formed in primary school and their ideas may not have grown significantly since then. This means they feel quite sure about two results:

(i) If you multiply two numbers the answer will always be bigger than either of the numbers you started with.

\[e.g. \quad 7 \times 8 = 56\]

(ii) If you divide one number by a second number (the divisor) the answer will always be smaller than the number you started with.

\[e.g. \quad 12 \div 3 = 4\]

The trouble is that this is only true if both the numbers being multiplied are greater than one, and if the divisor is greater than one. As soon as students work with decimals smaller than one they find that the false rule on which they have relied for so long breaks down. Trade calculations use a lot of numbers smaller than one, especially since the introduction of metricalation, and it may be that the
trade teacher is the person who has to help the students to develop their concepts to cope with confusing results like:

\[
\begin{align*}
7 \times 0.8 & = 5.6 \\
12 \div 0.3 & = 40
\end{align*}
\]

Some practical suggestions for dealing with this difficulty are:

(i) Use well known materials like money, timber, pipes and fabric with hands on exercises. \(7 \times 0.08\) is the same as ‘find the cost of 7 screws at 8c each’. \(12 \div 0.3\) is the same as ‘find how many 0.3m lengths of wood (or pipe) can be cut from a 12 metre length’. The lengths can be marked out with chalk to show how sensible the answer is.

(ii) Use a calculator to go through a sequence like:

\[
\begin{align*}
7 \times 8 & = 56 \\
7 \times 0.8 & = 5.6 \\
7 \times 0.08 & = 0.56
\end{align*}
\]

and in division show that the answers grow larger as the divisor gets smaller.

\[
\begin{align*}
12 \div 3 & = 4 \\
12 \div 0.3 & = 40 \\
12 \div 0.03 & = 400 \text{ and so on.}
\end{align*}
\]

(iii) Find practical examples specific to the trade and work through them with real materials. Encourage students to talk about what is happening in their own words. Then write down the processes in symbols and calculate the answers. Discuss how the results match the trade situation.
Worksheet 1.1

Calculations with Decimals

Addition
1. 0.8 + 0.6
3. 4 + 0.9
2. 0.094 + 0.97
4. 900 + 0.062 + 7.1

Subtraction
1. 1.2 – 0.6
3. 700 – 0.07
2. 97.6 – 9.76
4. Subtract 9.36 from 9.92

Multiplication
1. 0.08 × 4
3. 6 × 3.008
2. 3.4 × 9
4. 20 × 0.7

Division
1. 11.8 ÷ 2
3. 10.25 ÷ 5
5. 2 ÷ 8
7. 17 ÷ 6 (correct to 2 decimal places)
2. 0.35 ÷ 5
4. 0.4 ÷ 5
6. Divide 2 by 5
8. 0.02 ÷ 7 (correct to 3 decimal places)

Answers

Addition
1. 1.4
3. 4.9
2. 1.064
4. 907.162

Subtraction
1. 0.6
3. 699.93
2. 87.84
4. 0.56

Multiplication
1. 0.32
3. 18.048
2. 30.6
4. 14

Division
1. 5.9
3. 2.05
5. 0.25
7. 2.83
2. 0.07
4. 0.08
6. 0.4
8. 0.003

Worksheet 1.2

Hardest calculations with Decimals

**Multiplication**
1. How many decimal places are there in these decimals?
   (a) 24.56  (b) 24  (c) 0.999  (d) 23.5
2. Find the products:
   (a) 0.3×0.3  (b) 2.01×0.6  (c) 0.08×0.05
   (d) 0.2×0.4×0.6  (e) 2.3×4×0.3  (f) (1.2)^2

**Division**
1. How many places does the decimal point have to move to turn these numbers into whole numbers?
   (a) 0.25  (b) 1.2  (c) 0.006  (d) 0.0015
2. Move the decimal points to start these exercises and place the point in the correct place in the answer line, but do not do the calculation.
   (a) 0.5) 0.65  (b) 0.3) 48  (c) 0.12) 3.96
3. Do these divisions:
   (a) 0.6) 3  (b) 0.8) 0.64  (c) 0.02) 0.96
   (d) 0.4) 5  (e) 1.1) 0.0033

Answers

**Multiplication:**
1. (a) 2  (b) 0  (c) 3  (d) 1
2. (a) 0.09  (b) 1.206  (c) 0.004  (d) 0.048
   (e) 2.76  (f) 1.44

**Division:**
1. (a) 2  (b) 1  (c) 3  (d) 4
2. (a) 5) 6.5  (b) 3) 480.  (c) 12) 396.
   (a) 5  (b) 0.8  (c) 48  (d) 12.5
   (e) 0.003

Worksheet 1.3

Addition with Decimals

1. 0.3
   0.1
   +0.5

2. 0.4
   0.7
   +0.6

3. 5.3
   1.6
   +0.9

4. 24.9 + 4.05

5. 17 + 2.3

6. 5.047 + 1.55 + 2.6

7. 0.8 + 6 + 4.4

8. 0.0843 + 0.0099

Answers

1. 0.9
2. 1.7
3. 7.8
4. 28.95
5. 19.3
6. 9.197
7. 11.2
8. 0.0942

Worksheet 1.4

Subtraction with Decimals

1. \[ 8.7 - 2.5 \]  
2. \[ 8.5 - 4.7 \]  
3. \[ 9.07 - 3.47 \]

4. \[ 9.4 - 2.06 \]  
5. \[ 12.09 - 3.5 \]  
6. \[ 0.723 - 0.665 \]

7. \[ 4.231 - 0.58 \]  
8. \[ 150 - 0.9 \]

9. \[ 63.6 - 5.62 \]

Answers

1. \[ 6.2 \]  
2. \[ 3.8 \]  
3. \[ 5.60 \] or \[ 5.6 \]

4. \[ 7.34 \]  
5. \[ 8.59 \]  
6. \[ 0.058 \]

7. \[ 3.651 \]  
8. \[ 149.1 \]  
9. \[ 57.98 \]

Worksheet 1.5

Multiplication with Decimals

Place the point correctly in each answer.
Use extra zeros if necessary.

1. \(0.4 \times 2\) = 8
2. \(0.37 \times 5\) = 185

3. \(0.6 \times 0.24\) = 144
4. \(3.6 \times 2.5\) = 90

5. \(0.54 \times 10\) = 540
6. \(100 \times 0.234\) = 234

7. \(7 \times 0.001\) = 7
8. \(0.001 \times 27.50\) = 2750

9. \(0.27 \times 0.27\) = 729
10. \(0.06 \times 500\) = 3000

Answers
1. 0.8
2. 1.85
3. 0.144
4. 9.0
5. 5.40 or 5.4
6. 23.4
7. 0.007
8. 0.0275 or 0.02750
9. 0.0729
10. 30 or 30.00

Worksheet 1.6
Division with Decimals

Find decimal answers for each of the following:

1. \(25.5 \div 5\)
2. \(2 \overline{0.064}\)
3. \(3 \overline{0.15}\)

4. \(8 \overline{16.24}\)
5. \(\frac{18}{4}\)
6. \(4.2 \div 8\)

7. \(\frac{55}{100}\)
8. \(0.08 \div 10\)
9. \(0.2 \overline{0.6}\)

10. \(0.4 \overline{0.088}\)
11. \(\frac{3.6}{0.09}\)
12. \(0.5 \overline{0.7}\)

13. \(0.13 \div 0.3\) (correct to 3 decimal places)

Answers

1. 5.1
2. 0.032
3. 0.05
4. 2.03
5. 4.5
6. 0.525
7. 0.55
8. 0.008
9. 3
10. 0.22
11. 40
12. 1.4
13. 0.433

Chapter 2: Ratios and fractions

There have been widespread discussions in teaching circles about the place of fractions now that we have calculators. Decimals are certainly more commonly used for writing answers to calculations and measurements are more easily recorded with decimals in the metric system, while fractions were more simple for the imperial system. However, it is still essential for all users of mathematics to have a concept of fractions and to understand the way they are written down as common fractions, decimals or ratios. The most significant concept for all students to grasp is 'equivalent fractions'. In the past this concept was taught and practised briefly before the teacher went on to adding and subtracting fractions, during which the equivalent fraction ideas were continually being reinforced. Nowadays it is hard to justify the time and effort needed to teach addition and subtraction of fractions so it is really important to give work on equivalent fractions a lot of time in the curriculum.

In several trades such as Fitting and Machining there is a significant portion of the course which uses equivalent fractions or equivalent ratios, both in simplifying (e.g. $\frac{15}{20} = \frac{3}{4}$) or in expanding (e.g. $\frac{2}{5} = \frac{14}{35}$).

A few suggestions for giving meaning to working with fractions and ratios may be helpful to teachers in the engineering and automotive trades.

Ratios or fractions?

There are some good reasons why the difference between fractions and ratios exists. For example a ratio can express a relationship between three numbers, as in mixing concrete in a 1:3:4 mix. Fractions can only show a relationship between two numbers. Where possible it is a good idea not to fuss about these differences, but to use the words 'fraction' and 'ratio' interchangeably.

1 Common fraction ideas

Initial concepts

Many adults never use more complicated fractions than $\frac{1}{2}$ and $\frac{1}{4}$. These occur in
telling the time and in sharing things equally, such as food portions. The best shape to use to explain fractions is a circle as it can be cut into any number of equal parts by drawing radii and one or two missing parts show up immediately as the shape is clearly incomplete. This does not happen when using squares or rectangles. However the most handy teaching aid for quick explanations of difficulties is a piece of paper, folded to give halves, quarters, eighths and so on. It is wise to check that a student who is having trouble can explain the concept of a fraction or ratio with a concrete object like a piece of paper because it is known from educational psychology that these concepts are not well formed until the middle teen years. Many TAFE students may have incomplete fraction concepts.

Two meanings for the same fraction

Fractions grow out of division, because wholes are divided into equal parts. However there is another way to look at a fraction. This is linked to the concept of sharing, where several wholes are shared in equal lots. Consider the fraction $\frac{3}{4}$. In almost all school textbooks and in many TAFE classrooms you would find a sketch being used to explain this fraction. Perhaps there would be a circle cut into four pieces with three of them shaded.

True enough, this is one meaning of three-quarters. It is also necessary to give an equal amount of attention to the other meaning, which can be shown by taking a piece of string three metres long and folding it in half and in half again to give four pieces. If you cut them, each one is a quarter of the 3m you started with and each, when measured, is 750mm or $\frac{3}{4}$ of a metre long.

So $\frac{3}{4}$ can mean one whole, cut into four equal parts, with three put together to give you three quarters; it can also mean three wholes being shared or divided into four equal lots, each lot being a quarter of three.

$$\frac{3}{4} = 3 \times \frac{1}{4} \quad \text{or} \quad \frac{3}{4} = \frac{1}{4} \times 3$$
The reason for emphasising the two meanings of a fraction is that the second way of looking at fractions is the better one for teaching students that the division operation is being done by the 4; that the meaning of \( \frac{3}{4} \) is 3 divided by 4, not 4 divided by 3. Understanding this will enable them to change fractions to decimals using a calculator by pressing the right buttons.

When mathematics tests start to take notice of the widespread use of calculators in industry, schools, shops and homes there should be changes in the type of items written to test knowledge of fractions and decimals. Such a question might be:

*Which of these is the correct way to change \( \frac{3}{4} \) to a decimal: \( 3 \div 4 \), or \( 4 \div 3 \)?*

It should not be assumed that TAFE trade students will have an awareness of both these meanings for a fraction. Teachers know that many students are not able to change fractions to decimals using the correct program, and that they seem unable to check the reasonableness of the decimal equivalent which they have found. How can \( \frac{3}{4} \) be equal to 1.3333?

## 2 Equivalent fractions

A chart of multiples is very handy for explaining equivalent fractions. Displayed on an overhead projector, or handed out on separate pages to each student, the chart looks like a full set of multiplication tables facts.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
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<th>5</th>
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<td>48</td>
<td>56</td>
<td>64</td>
<td>72</td>
<td>80</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>18</td>
<td>27</td>
<td>36</td>
<td>45</td>
<td>54</td>
<td>63</td>
<td>72</td>
<td>81</td>
<td>90</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
<td>100</td>
</tr>
</tbody>
</table>

This is not just the ‘tables’, but a simple teaching aid for equivalent fractions. If you look at the top two lines not as two rows of numbers but as one row of fractions you will see a set of equivalents for \( \frac{1}{2} \).

\[
\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \ldots
\]
Any pair of lines gives its own set of equivalent fractions, starting with \( \frac{5}{6} \), or \( \frac{7}{8} \), or \( \frac{9}{10} \) for example. Students can find more fractions by extending the patterns of multiples on past 10 times to 11, 12 and even 15 times, for they will meet large numbers in topics like gear ratios. More varieties of fractions can be obtained by folding one line down past several others, say bringing line 2 down to line 5. This gives many more possibilities like:

\[
\frac{2}{5}, \frac{4}{10}, \frac{6}{15} \text{ etc.}
\]

Use of the chart reinforces the fact that all the equivalent fractions have come from the multiplication tables.

### 3 Simplifying fractions

Simplifying fractions now becomes a game of finding any smaller number which will divide evenly into both the numbers of the fraction. It is not necessary to find the biggest such number immediately. When simplifying a fraction like \( \frac{42}{56} \), it is not wrong to divide both numbers by 2 first and then by 7, even though in the old days we used to be expected to find the highest common factor (HCF) first and then divide by 14 to get \( \frac{3}{4} \) in one step. The word ‘factor’ for the numbers used to divide both numerator and denominator should not cause problems as it is common terminology from primary and secondary school.

The chart of multiples gives students a starting point for finding suitable factors when they have to simplify fractions. With \( \frac{42}{56} \), they can find 42 and 56 in the column that is headed by 7, so division by 7 is right. The resulting fraction \( \frac{6}{8} \) obviously can be simplified to \( \frac{3}{4} \). Many students will want to write out what they are doing as follows, rather than using ‘cancelling’.

\[
\frac{42}{56} = \frac{42 \div 7}{56 \div 7} = \frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}
\]

At least these students show that they know what they are doing. In time they will probably speed up. It is better to wait until they realise for themselves that the middle steps are not necessary, rather than the teacher giving them short cuts. When they shorten the work themselves it is an indication that they understand.

For many students the chart of multiples is an eye-opener to the patterns on which the equivalent fractions are based. They should respond positively to something which makes sense of processes which may in the past have been very hit and miss.
4 Applying equivalent fractions

Frequently in trade applications workers have to choose from a limited set of numbers to expand a ratio. For example, in an engineering workshop, a dividing head may be used to mark out exactly the positions of teeth on a gear. The required ratio might be \( \frac{2}{3} \). Several metal plates would be available with a certain number of holes on each. The worker has to find a plate which has a number divisible by 3. (The 3 is found from the denominator of the fraction \( \frac{2}{3} \).)

A common set of number hole circles has these numbers of holes:

15, 16, 17, 18, 19, 20, 21, 23, 27, 29, 31, 33, 37, 39, 41, 43, 47, 49

Starting at the biggest, 49, the worker has to divide each number in turn by 3 until he or she comes to one which divides evenly. When 39 is reached the answer is 13 exactly, so the 39-hole plate is the one to use and 13 is the factor needed to create an equivalent fraction to \( \frac{2}{3} \). Many students would write their calculation out this way:

\[
\frac{2}{3} = \frac{2 \times 13}{3 \times 13} = \frac{26}{39}
\]

The new fraction means that the crank handle will have to be turned to the 26th hole on the circle with 39 holes.

In time, students will make the choice of the correct number hole circle more and more quickly, as the sets of circles are standard and the work becomes routine.
Estimation is an important life skill. The ability to estimate a length, area or quantity of material with a reasonable degree of accuracy is essential for anyone working in a trade area.

The widespread use of calculators in trade mathematics further increases the need for estimating. Calculator users need to develop a feeling for what the answer should be, and be alert for mistakes in key-stroking or malfunctions of their calculator. Worksheet 3.1 is a suggested way of giving students practice in actually using a calculator. No more details of how to use a calculator are given in this handbook. Recent changes in secondary schools have made the use of a calculator a standard part of the syllabus. Good teaching materials can be found in school textbooks.

Estimating can be divided into three main areas:

1. Estimation using number skills.
2. Estimation using mathematical commonsense.
3. Estimation using concepts of size.

Sometimes only one type of estimate can be made, but often two, or all three are used in the same calculation.

Estimation skills need to be taught. Students can only become confident estimators if they are given opportunities to practise estimating and to compare their estimates with exact answers or actual measurements.

Calculators will perform operations on numbers accurately, but the answer that the calculator gives may not be the right answer to the given problem. The student may have chosen the wrong operation, pressed the wrong key or missed out a step in the calculation. Before working out a calculation, students should always make an estimate of the size of the answer to be expected. This is especially important when working with decimals, as it helps to fix the position of the decimal point. Students should be encouraged to look critically at the answer that is displayed on the calculator to see if it is close to their estimate. In this way, estimating will reveal slips or careless errors.

1 Estimation using number skills

To estimate the results of a calculation before it goes on the calculator, students need to be able to do the following:
(i) Round off numbers to the nearest whole number, 10, 100, etc.

\[ \text{e.g. 287 rounded to the nearest 10 is } 290 \ \text{Answer: 290} \]

(ii) Handle multiplication and division with zeros.

\[ \text{e.g. } 300 \times 79, \ 1000 \div 50 \]

Skills (i) and (ii) are essential. Skills (iii) and (iv) are extra ones for 'smooth operators'.

(iii) Choose the round numbers that make the estimating easy to do.

(iv) Decide if the estimate is likely to be a bit bigger or a bit smaller than the exact answer.

Estimation Worksheets 3.2, 3.3 and 3.4 were designed to help students develop the skills they need for estimation. Worksheet 3.2 is a pre-test that could be used to check what skills students already have. Worksheets 3.3 and 3.4 contain graded and fairly structured exercises designed to teach the component skills and to build confidence in using them. Worksheet 3.4 involves applying the skills developed in Worksheet 3.3 to more complex calculations.

Comparing the estimate with the exact answer gives important feedback when learning to estimate. It proves that the process really works. In Worksheet 3.4 however, the point is made that if obtaining an estimate is not possible or too difficult, students should be encouraged to check their work by simply repeating the calculation. This is meant to be a discussion worksheet where the processes are more important than the answers. If worksheets like these are used, it is a good idea to follow them immediately with some trade specific examples.

A great advantage of teaching students to do this kind of estimate before using their calculator is that they go through the steps of the problem, but with easy numbers. This should help students realise that there is no magic about the answer the calculator gives. Using a calculator is just a quicker way of working out a problem than doing it on paper.

Note: trade practices

Many trades have their own practices for rounding answers up, and occasionally down, at the end of a calculation.

- When buying timber, lengths have to be rounded up to the next multiple of 0.3m starting from a 1.8m length.
- Amounts of concrete have to be bought in multiples of 0.2m$^3$.
- If you need 4.843 litres of paint, you will have to buy five litres.
- To find out how many 850mm lengths can be cut from a 6m length, you will need to round down.
- A shaft which has a diameter of 26.25mm, has a nominal diameter of 26mm.
- If you have a current of 8.33 amps, you need a 10 amp fuse.
- If a customer's bill comes to $43.796, you charge $43.80, or perhaps $45!
Estimation using mathematical commonsense

Sometimes it is possible to get a fairly accurate estimate by just considering the problem. Often the upper and lower limits for an answer are obvious.

For example, when using Pythagoras’ Theorem to work out the length of the hypotenuse of a triangle, there is an absolute upper limit on the length of the hypotenuse in relation to the other two sides. The four examples below make the point.

The length of the hypotenuse is:

1. Just less than \(1\frac{1}{2} \times \text{one of the other sides}\)
2. Exactly \(1\frac{1}{4} \times \text{the longer side}\)
3. Approximately \(1\frac{1}{6} \times \text{the longer side}\)
4. Equal to the longer side + a bit

Two conclusions that can be made about the length of the hypotenuse are:

(i) The longest it can be is nearly \(1\frac{1}{2} \times \text{one of the other sides}\). This happens when the other two sides are equal.
(ii) In a very thin triangle, it will only a bit bigger than the longer side.

Using this knowledge, the student can easily work out the upper and lower limits for a reasonable answer and which value it will be closest to.
There are other obvious limits that an experienced person puts on his/her answer that less experienced students probably do not think of, for example, knowing that the area of a circle has to be less than the diameter squared or even better, knowing it will be approximately \( \frac{3}{4} \) of the diameter squared. Applying the knowledge of what happens to the answer when dividing by a number less than one (see Decimals pages 8–9) will also alert students to mistakes.

There are occasions when a fairly accurate estimate can be made by carefully looking at a diagram.

For example, look at the typical item below.

![Diagram showing a circle with diameters labeled as \( \phi_{30} \) and \( \phi_{48} \).]

What is the width ‘\( w \)’ of the flats on the component shown in the diagram?

Clearly the answer has to be between 30 (the shorter diameter) and 48 (the longer diameter).

In these sort of cases, there is often no need to make the kind of estimate outlined in section 1 of this chapter.

The best way to encourage students to use commonsense when estimating is to talk with them about it. Worksheet 3.5 gives examples that could be used as a basis for discussion about estimating percentages. Asking students how they would estimate each of the items will probably reveal many different methods. In this way students can learn from each other and probably teachers can pick up a few ideas too.

3 Estimation using concepts of size

When students are working out problems which involve sizes, an estimate can often be made by using their knowledge of sizes of common objects. However, in order to do this, they need to have clear concepts of size. Experienced trades people have developed the ability, for example, to look at a wall and estimate its area, to look at a tank and have a good idea of its capacity and so on. This ability comes from working constantly using the relevant measurements. Young students are unlikely to have this ability, particularly pre-apprenticeship students who are not getting ‘on the job’ experience. Concepts of size therefore need to be taught.

The first step is to make sure students have a clear idea of the metric system. A
good technique is to show them everyday things that will give them a clear image of the various units. These can then be used to work out estimates of actual objects. Some suggestions are:

**Linear measurements**

- **millimetre:** width of the tip of a ballpoint pen
- **centimetre:** width of a person’s little finger, or finger nail
- **metre:** Have the students use a metre ruler to measure one metre from the floor up the side of their body.
- **kilometre:** Suggest students use the trip meter in a car to measure one kilometre from their home along a familiar route.

**Square measurements**

- **hectare:** two soccer-fields side by side.
- **square metre:** A useful teaching aid is a cubic metre kit consisting of 12 metre rods with corner joiners. Making a square metre first allows students to see just how big it is. If square metres are a frequently used measurement, outlining a square metre on a wall will allow students to become familiar with it and help them when they attempt to estimate areas.
- **parts of a square metre:** If students are often asked to work out very small areas in terms of square metres, it is a good idea to mark out typical areas within the square metre and label them. For example, mark out an area 10mm $\times$ 10mm and label it 0.0001m$^2$. Students can see how many decimal places are involved in a typical answer and they will learn to expect very small decimals for small areas.

*Note:* Not many students will have had much practice in working with decimals of more than three places. Making the connection between the number of 10mm squares and the area expressed in square metres will reinforce some decimal concepts. There is a paradox here about decimal notation: a $10 \times 10$mm square fits 10 000 times into a square metre and its area is 0.0001m$^2$. Ten thousand, 10 000, has four zeros. One ten-thousandth (0.0001) has only three essential zeros.

**Cubic measurements**

- **cubic metre:** Fully assembling the cubic metre kit gives students a clear idea of its surprisingly large size. One suggestion for explaining how far a cubic metre of concrete spreads is to build a cubic metre with ten 100mm thick square metres of foam which can then be spread over the floor.

  Again, if students have to work out very small volumes in terms of cubic metres, placing a 10mm cube inside a cubic metre and labelling it 0.000 001m$^3$ gives them a standard to work from.
Relationship between capacity, volume and mass

A cubic metre kit is an excellent basis for explaining the relationships within the metric system. Many students have not realised how simple the system is and working with the kit helps it to make sense.

The 100mm cube inside an assembled cubic metre can be used to make many connections. For example, a litre of water has a mass of a kilogram and fills a 100mm cube. A litre of water poured inside a cubic metre will be 1mm deep. 1000 litres of water would fill it and the filled cubic metre would weigh a tonne.

The 100mm cube is also useful in explaining units of pressure. Filled with water, it exerts a pressure of nearly 1kPa on a surface. Students can hold it so they can actually feel a pressure of 1kPa. (Force due to gravity must be taken as approximately 10N.)

Once the students have clear concepts of size they need practice in estimating and checking their estimate against actual measurements. This will help them to become confident in their ability to estimate.

It will also help if they know the average size of things they commonly work with, for example, the floor area of an average bedroom, the height of an average door. When smaller objects are involved, such as a cylinder, labelling one with its cross-sectional area and capacity will give students a reminder of what a reasonable answer should be. Of course, using this type of estimating will only help students to work out problems if teachers use realistic measurements when they write questions.
Worksheet 3.1

Using a Calculator

Explanation

This worksheet aims to teach students to use their calculator correctly. An unusual feature of the worksheet is that the answers are given next to the calculations so that the effort goes into getting the correct answer, rather than just getting an answer. If students are using their calculator incorrectly, they will know straight away; they will have to try again and again until they come up with the correct result.

Items 1–4 help pick up careless errors. Encourage students to glance at the display before pressing the next button. Check that they know the use of C and CE buttons.

A calculator can be used to allow students to grasp mathematical concepts for themselves. Items 5–8 help them understand the relationship between squares and square roots.

Rounding numbers can cause problems for some students. In Item 11, they will have to be able to round the answer to an appropriate number of decimal places. Note that some calculators will give the answer as 0.16666667.

Students need to know order of operations to complete Items 12–16. Item 16 in particular often causes problems. The most common error is to put into the calculator: \( 10 - \pi \times 62 \). Here especially, because the answer is provided, students will know immediately that something is wrong, and will either solve it for themselves or ask for help.

## Worksheet 3.1

**Using a Calculator**

Use your calculator to do these calculations. Check your answer with the answer given.

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $10.21 + 12.4026$</td>
<td>$22.6126$</td>
</tr>
<tr>
<td>2. $18.264 - 4.021$</td>
<td>$14.243$</td>
</tr>
<tr>
<td>3. $17.26 \times 13.5$</td>
<td>$233.01$</td>
</tr>
<tr>
<td>4. $733346.91 \times 0.0007069$</td>
<td>$518.4029307$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. $42^2$</td>
<td>$1764$</td>
</tr>
<tr>
<td>6. $\sqrt{1764}$</td>
<td>$42$</td>
</tr>
<tr>
<td>7. $0.025^2$</td>
<td>$6.25 \times 10^{-4}$ or $0.000625$</td>
</tr>
<tr>
<td>8. $\sqrt{0.0007243}$</td>
<td>$0.26912822$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>9. Change $\frac{12}{32}$ to a decimal</td>
<td>$0.375$</td>
</tr>
<tr>
<td>10. Change $\frac{24}{5}$ to a decimal</td>
<td>$4.8$</td>
</tr>
<tr>
<td>11. Change $\frac{1}{6}$ to a decimal</td>
<td>$0.1666666666$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. $\frac{2 \times 3^2}{6}$</td>
<td>$3$</td>
</tr>
<tr>
<td>13. $\frac{(2 \times 3)^2}{6}$</td>
<td>$6$</td>
</tr>
<tr>
<td>14. $\frac{300 \times 30}{75}$</td>
<td>$120$</td>
</tr>
<tr>
<td>15. $\frac{\pi \times 0.021^2}{4}$</td>
<td>$3.4636059 \times 10^{-4}$ or $0.00034636$</td>
</tr>
<tr>
<td>16. $\frac{10}{\pi \times 62}$</td>
<td>$0.051340304$</td>
</tr>
</tbody>
</table>

Choose the answer most likely to be correct for each calculation. Do not actually work it out.

1. \[18.76 + 131.24 = ?\]
   a. 212  
   b. 150  
   c. 128

2. \[693.07 - 124.93 = ?\]
   a. 397.14  
   b. 818.21  
   c. 568.14

3. \[12\frac{1}{2}\% \text{ of } 489.60 = ?\]
   a. $61.20  
   b. $6.12  
   c. $40.80

4. \[5.7^2 = ?\]
   a. 11.4  
   b. 324.9  
   c. 32.49

5. \[5.5 \times 2.64 = ?\]
   a. 145.20  
   b. 14.52  
   c. 18.520

6. \[200 \times 0.760 = ?\]
   a. 152  
   b. 1520  
   c. 15.2

7. \[3 \div 0.6 = ?\]
   a. 0.5  
   b. 50  
   c. 5

8. \[\frac{175 \times 1.6 \times 60}{90} = ?\]
   a. 186.67  
   b. 1866.7  
   c. 18.67

Answers

1. 150  
2. 568.14  
3. $61.20  
4. 32.49  
5. 14.52  
6. 152  
7. 5  
8. 186.67

### Worksheet 3.3

**Estimation Exercises 1**

1. Round off these numbers to the nearest 10.
   - 27
   - 35
   - 512
   - 7389
   - 4306

2. Round off these numbers to the nearest 100.
   - 423
   - 291
   - 3077
   - 550
   - 1983

3. Round off these numbers to the nearest whole number.
   - 7.2
   - 12.6
   - 0.8
   - 19.5
   - 49.7

4. Estimate, by rounding to the nearest 10.
   - **Example**
     - 103 + 68
     - 75 ÷ 18
     - 180 − 21 − 56
     - 26 × 13
   - **Round Nos**
   - **Estimate**

5. Estimate, by rounding to the nearest 100.
   - **Example**
     - 593 + 8763
     - 5862 − 2097
     - 479 × 234
     - 7752 ÷ 170
   - **Round Nos**
   - **Estimate**

6. Estimate, by rounding to the nearest whole number.
   - **Example**
     - 2.7 + 7.9
     - 19.2 − 6.81
     - 5.3 × 4.9
     - 9.96 ÷ 1.8
   - **Round Nos**
   - **Estimate**

7. Use your calculator to find the exact answers and compare them to the estimates.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>30</td>
<td>40</td>
<td>510</td>
<td>7390</td>
</tr>
<tr>
<td>2.</td>
<td>400</td>
<td>300</td>
<td>3100</td>
<td>600</td>
</tr>
<tr>
<td>3.</td>
<td>7</td>
<td>13</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>4.</td>
<td>170</td>
<td>4</td>
<td>100</td>
<td>300</td>
</tr>
<tr>
<td>5.</td>
<td>9400</td>
<td>3800</td>
<td>100000</td>
<td>39</td>
</tr>
<tr>
<td>6.</td>
<td>11</td>
<td>12</td>
<td>25</td>
<td>5</td>
</tr>
</tbody>
</table>

Worksheet 3.4

Estimation Exercises 2

1. **Estimate and calculate:**
   - **Step 1** Write round numbers for each example.
   - **Step 2** Use round numbers to find estimates.
   - **Step 3** Decide if the estimate will be too big or too small.
   - **Step 4** Calculate the exact answers.
   - **Step 5** Compare estimates and exact answers.

<table>
<thead>
<tr>
<th>Examples</th>
<th>Round Nos</th>
<th>Estimate</th>
<th>Big or Small</th>
<th>Exact Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $49.93 \div 257.68 + 81.76$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. $18.2 - 7.92 - 0.4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. $\frac{49.3 \times 24.5}{6.81 \times 4.7}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. $20.976 \div 3.71$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. **How would you estimate the following?**
   - a. $0.49 \times 7.18$
   - b. $0.4088 \div 5.6$
   - c. $6.912 \div 0.15$
   - d. $0.00172 \div 0.0144$
   - e. $2.7^2 + 1.9^2$
   - f. $\sqrt{156}$
   - g. 7½% of $195.70$

3. **Would it have been better to do some of the items in Question 2 twice on a calculator, and not bother with estimating? Which items?**

Worksheet 3.5

Estimating Percentages

Discuss sensible ways to estimate the following percentages.

1. 12½% of $94

2. 5% of 15m

3. 7½% of 47m²

4. 3½% of $9.30

5. 15% of 480

6. 85% of 1,586 tonnes

7. 2½% of 121

8. 90% of 6.5m³

9. 80% of 48kg
Chapter 4: Teaching concepts

Why should we bother to teach concepts? Why not just teach the processes and the rules?

The answer is:

We teach concepts so that the rules and the processes will be —

• understood
• applied correctly, and
• remembered.

1 Knowledge is based on concepts

The aim of trade teaching is to produce good trades people who know their job. To know something in a trade context means that you learnt it well in the first place and practise it often so that you remember it. Trades people apply their knowledge to routine tasks and adapt it as circumstances change. The first step to knowledge is understanding the concept. If something is well understood from the beginning then you know —

• what it’s about
• what it all means
• why it’s done this way, and
• how it works.

2 A multi-sensory approach

Teaching concepts thoroughly requires students to be given experiences that involve far more than abstract thinking with words, diagrams, numbers, formulas and other symbols. Learning new concepts involves handling objects or models, feeling them, walking around to look at them from different directions, taking them apart and putting them back together. There are more senses operating than just sight and hearing. Students absorb ideas like size and mass, shape
and pattern. Through these experiences they may also grasp the restrictions of what can and cannot be done.

If students are given a learning-by-doing experience with a variety of activities including real objects and models, the chances are that their interest level will be very high and they will not forget that experience. The trouble is that the mathematical concept which you wanted them to learn may not be remembered. You have to help them to make the connections between the concrete experience and the way the concept is written down and used in calculations.

3 Linking the concrete to the symbolic

There have to be clear links between the learning experiences that introduce new concepts and the rules, formulas and calculations. The links begin to develop from the first time the concept is met and are transferred to the calculations only if the teacher is very careful at every step. Here is a way to proceed.

a. Show the model. Talk about it. Get the students to discuss and handle it and ask questions. Include everyone.

b. Without moving away from the model, write the formula and the calculations on some handy paper, e.g. on butcher's paper placed beside the model, until all the links are clear.

c. Take the paper to the chalkboard and record the formula and the calculations there. Keep the butcher's paper for future reference.

d. Go back to the model. Ask students to explain each part of the chalkboard calculations by touching the real object.

e. Give students calculations to do on their own paper, alone or in pairs and small groups. Meantime, move around the room to check for confusions until everyone can handle the process. Individual students can be shown the model or the butcher's paper again to strengthen understanding.

f. When everyone can explain each feature of the calculation, change to symbols only and do more examples for reinforcement.

Later, there will be students who will forget and make errors. They can go back to the model and repeat the steps. Clearly, concrete teaching aids have to be stored in a handy place so they can be used again and again quite easily. They need to be a sensible size and easily assembled. Some students will appreciate being able to revise the concept using the model, even months after they first changed to symbols.

Those students who missed out on the first lesson due to illness, say, should be allowed to work with the model until they can show that they have grasped the concepts. Otherwise they will blindly follow the rules for the calculations and be left wondering what it all means.

4 A bonus

Quite often when the teacher uses a model to teach a topic, like hip roofs and gable roofs, some of the students will realise that to them, it explains something quite different.
They will see that if it is turned upside-down it shows roof valleys, or that it could be easily changed to look like a trench with a trapezium cross-section. So the student performs the teacher’s role for a short while and creative learning takes place.

5 Outline of a lesson teaching a concept

Rationale
Many students confuse area and perimeter. Lessons which strengthen the two concepts have a valuable carryover to practical applications. In this lesson we explore the relationships between area and perimeter when one of them is a fixed quantity.

Procedure

Part A
Materials needed: Four small square tiles per student, sheets of 1cm grid paper.

Step 1: Hand out to students four tiles and a sheet of grid paper. Ask them to arrange all four tiles on the desk so that a full side of each tile touches a full side of another. One result might be:

Step 2: Check the arrangements around the room to see that the tiles are touching correctly. Students should then sketch the shapes they have made on the grid paper.

Step 3: Repeat this until they cannot find any more shapes. Every student should be given time to find three or four shapes. Altogether this should yield five shapes around the class:

Step 4: Record all the shapes on the chalkboard or an OHP transparency. Have the students record all the shapes on the grid paper. Establish, by discussion, that all the shapes have the same area.

Step 5: Ask students to count around the shapes and write down the perimeter of each. One side of a square counts as one unit. (The answers are 10 units for four of the shapes, but the square’s perimeter is eight units.)

Step 6: Pose the question, ‘If different shapes have the same area, do they also have the same perimeter?’ Discuss this.

Step 7: Record on the chalkboard: Shapes which have the same area do not all have the same perimeter.
Part B

Give the students this problem to solve.

I have 36m of fencing to use around a vegetable
garden in the middle of the backyard. I want to
have a square or rectangular garden. What are
the possible lengths and breadths I can use?
(Do not use parts of a metre.)

Materials needed: A piece of string about five metres long, sheets of 1cm grid
paper or dot paper.

Step 1: Tie the ends of the string together and use two students to hold it so that it
becomes a rectangular shape. This is a model of the 36 metres of fencing. Ask the
students to sketch one shape on the grid paper. One possible shape might be:

```
  12
 /   \
 |    |
 |    |
 /   \
  6   6  P = 36
 /   \
 |    |
 |    |
 /   \
  12
```

Step 2: Check the answers around the room to see that no one is using numbers that
multiply to give 36 by mistake, e.g. 9 and 4. Correct this error by discussion.

Step 3: Using grid paper, ask students to draw as many different shapes as they can to
fit the problem. Every student should be able to find several shapes. Collect all answers
around the class. The full range of possibilities is:

<table>
<thead>
<tr>
<th>Length</th>
<th>Breadth</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
</tr>
<tr>
<td>13</td>
<td>5</td>
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<td>12</td>
<td>6</td>
</tr>
<tr>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

Step 4: List the answers in order on the chalkboard. Have the students draw all the
shapes they missed on grid paper until everyone has nine sketches.

Step 5: Discuss the original situation: all the shapes use the same length of fencing. Ask
students to record inside each sketch the area of the rectangle or square. Record these
in a column beside the length and breadth columns already on the chalkboard.

Step 6: Ask the following questions:
   a. Do all the shapes have the same perimeter?
   b. Do all the shapes have the same area?

Discuss the results.

If time allows, patterns can be found in the sequence of answers in the area column.
Reasons for this could be explored.

Step 7: Record on the chalkboard:
Shapes which have the same perimeter do not all have the same area.
Part C

Discuss the special case of the square. When the area was fixed the square had the smallest perimeter. When the perimeter was fixed the square had the largest area. Apply this to real-life situations like having the greatest space available for growing vegetables and designing economical house-plans.

This is a simple lesson. It can be presented in a short space of time. On the other hand, it can take the form of an unusual homework assignment. See Worksheet 4.1: Homework Assignment.
Worksheet 4.1

Homework Assignment

1. Take four square tiles or pieces of cardboard of any small size. Let the length of a side equal one unit. Arrange the four squares so that a full side of one touches a full side of another and find the perimeter.

   e.g. \[\boxed{\phantom{1000}} \quad P = 10 \text{ units}\]

   Arrange them again in as many ways as you can.
   Draw a sketch of each new shape you get and write under each sketch the perimeter, as above.
   Keep going until you have five different shapes.

   Questions:
   1. Do all the shapes have the same area?
   2. Do all the shapes have the same perimeter?
   3. Which shape has the smallest perimeter?
   4. Is this true or false, ‘all equal areas have the same perimeter?’

2. If you had a piece of light fencing you could bend it to make a rectangle or a square. Imagine the fencing is 36m long and the sides have to be exact metre lengths, e.g. a rectangle, 10m long and 8m wide. Find the area in \(m^2\) and write this inside the rectangle.

\[\boxed{8 \times 10} \quad A = 80m^2\]

Find as many shapes as you can. Sketch each shape, label the lengths of the sides and calculate the area.

Questions:
   1. Do all the shapes have the same perimeter?
   2. Do all the shapes have the same area?
   3. Which shape has the largest area?
   4. Is this true, ‘Every rectangle or square with perimeter 36m has the same area?’

Many teachers comment that their students cannot do transposition of formulas. There are several reasons why this is such a difficult area for students, including:

- terminology, and
- the place of formulas in school mathematics.

1 Terminology

The words ‘transposition’ and ‘transpose’ are not used in mathematics teaching before students come to TAFE. They have only rarely appeared in a school mathematics textbook published since the 1940s. Here are the words which have replaced them:

- ‘transposition’ has become ‘change of subject’;
- ‘transpose for y’ has become ‘change the subject to y’.

There are two ways in which this change is seen as an improvement. Firstly, maths is taught as a language. Before using formulas students work with sentences, e.g.

\[
\begin{align*}
5 + \square &= 17 & & \text{and} \\
3a + 7 &= 19
\end{align*}
\]

So a formula like the area of a trapezium is seen as a sentence:

\[
A = \frac{1}{2}(a + b)h \\
A \text{ is the subject} \\
= \text{is the verb}
\]

If it is changed to:

\[
h = \frac{2A}{a + b}
\]

then ‘h’ has become the subject of the sentence. This links up with other language
experiences. Compare these two sentences where the subject has been changed. (The subjects are in bold.)

That tall blond boy is the fastest runner.

The fastest runner is that tall blond boy.

Secondly, mathematics is taught for understanding, rather than by rules. Transposition means 'crossing over', but there is a lot more to changing the subject of formulas than just passing letters from one side to the other in a kind of special routine. The old rule, 'change sides, change signs', is inadequate to describe all the processes, but it can also be misleading. If students follow it slavishly they make the following type of error with the plus and minus signs:

\[
\frac{\pi}{2} P = a + b
\]

\[a - b = \frac{\pi}{2} P\]

If you try to set out rules for the steps to be followed in changing the subject of formulas it becomes very complicated and even a slightly different formula has to have a new method. The best thing is to teach methods based on understanding what is going on. Basically, at each step students learn:

to do the same mathematical process to both sides of a formula

These processes include:

- adding (or subtracting) the same number;
- multiplying (or dividing) by the same number;
- taking square roots of both sides;
- squaring both sides.

Many teachers like to use an actual beam balance, or sketches of a balance, to reinforce the idea of treating both sides equally. It is also useful to write the two sides of a formula on two separate pages of scrap paper, making sure that any change made to one page is also made to the other page.

2 The place of formulas in school mathematics

Formulas are only a small part of Year 7–10 mathematics. They are used as a tool for finding a value that is needed for a task. For example:

- finding the interest to pay on a loan;
- calculating the area of a plot of land;
- finding how many litres a tank holds.

Most school students do not do any other work with formulas. Change of subject of formulas is usually taught by a substitution method which gets rid of all the letters in the formula except the one that is needed. Students find it easier to work with numbers because they are less abstract than letters.
Example:
Find the height of the trapezium in this diagram. The area is 12000mm$^2$.

![Diagram of a trapezium with dimensions 200mm and 100mm]

**Numerical method**

\[ A = \frac{1}{2}(a + b)h \]
\[ 12000 = \frac{1}{2}(100 + 200)h \]
\[ 12000 = \frac{1}{2} \times 300 \, h \]
\[ 12000 = 150 \, h \]
Divide both sides by 150:
\[ \frac{80}{750} = \frac{h}{750} \]
\[ 80 = h \]
\[ \therefore h = 80 \]
The height is 80mm.

**Traditional method**

Change of subject of formulas in which letters are moved about before numbers are inserted is only taught to students of the very best ability, less than one quarter. Here is the way the above example would be done.

\[ A = \frac{1}{2}(a + b)h \]

Multiply both sides by 2:
\[ 2A = (a + b)h \]
Divide both sides by \((a + b)\):
\[ \frac{2A}{a + b} = \frac{(a + b)h}{a + b} \]
\[ \frac{2A}{a + b} = h \]
\[ \therefore h = \frac{2A}{a + b} \]
\[ = \frac{2 \times 12000}{100 + 200} \]
\[ = \frac{24000}{300} \]
\[ = 80 \]
If students want to check back on their work they can read the lines:

- ‘multiply both sides by 2’ and
- ‘divide both sides by \((a + b)\)’.

These are like signposts showing the track which was taken. As the students have more practice the words are omitted and the example looks like this:

\[
A = \frac{1}{2}(a + b)h \\
2A = (a + b)h \\
\frac{2A}{a + b} = \frac{\frac{1}{2}(a + b)h}{a + b} \\
h = \frac{2A}{a + b}
\]

Rules have been replaced by a process that makes sense. All the steps are based on doing the same to both sides of an equation so that the verb, ‘is equal to’, continues to be true. Some people have succeeded by using rules, but rules will only work if the student is keeping up constant practice. The dangers are that rules do not transfer easily to new situations and that a half-remembered rule is of no value.

### 3 Teaching formulas in trade courses

Here is a typical method for changing the subject of a formula taken from a recent trade textbook.

\[
\text{grade} = \frac{\text{rise}}{\text{distance}} \\
i.e. \quad G = \frac{R}{D} \\
\therefore \text{by transposition}, \quad R = G \times D
\]

An alternative method is recommended because it follows on well from school methods.

\[
\text{grade} = \frac{\text{rise}}{\text{distance}} \\
G = \frac{R}{D} \\
\text{Multiply both sides by } D \\
G \times D = \frac{R}{D} \times \frac{Q}{1} \\
G \times D = R \\
\therefore R = G \times D
\]

Another formula which is common to many trades is Pythagoras’ Theorem.
The span of a roof is 10 metres with a rise of 2.5 metres. Find the length of the common rafter.

The usual trade teacher’s way to do this question is:

\[ CR = \sqrt{\frac{1}{2} \text{span}^2 + \text{rise}^2} = \sqrt{\frac{1}{2} \cdot (10)^2 + 2.5^2} = \sqrt{25 + 6.25} = \sqrt{31.25} = 5.6 \]

**Notes**

Students may have difficulty because:

1. The formula does not look like Pythagoras’ Theorem as they were taught it at school, \( c^2 = a^2 + b^2 \).
2. In algebra, \( CR \) means \( C \times R \) and \( \frac{1}{2} \text{span}^2 = \frac{1}{2} \times (\text{span})^2 \). It should be written as \( \frac{1}{2} (\text{span})^2 \).
3. The square root is over a very complicated set of numbers.

The method which would be used by a student who was following his school mathematics is:

\[ CR^2 = 5^2 + 2.5^2 = 25 + 6.25 = 31.25 \]
\[ \therefore CR = \sqrt{31.25} = 5.6 \]

**Notes**

1. A new triangle is drawn to show the needed lengths.
2. The equation is easily recognised as Pythagoras’ Theorem.
3. The square root is used over one number only.

### 4 Summary

(a) The words ‘transposition’ and ‘transpose’ are strange new words to the majority of students.
(b) Most students have never worked with formulas in which the letters are moved around. The numerical method is much more common.
(c) Change of subject should be explained by the method of doing the same process to both sides of an equation and not by rules.
(d) TAFE lessons with formulas should build on established school methods wherever possible.
Many trade students do not seem to be able to apply their mathematical skills and practical trade knowledge to written calculations. They either take one look at a written problem and decide they can’t do it or start working with the first few numbers they come across without fully reading the question.

Many teachers experience varying degrees of concern, frustration and annoyance when their students, having successfully tackled a practical problem in the workshop, seem prepared to accept impossibly large or small answers to related written calculations, or hand in pages of muddled working.

It is important to try and analyse the reason for such student difficulties with written calculations and not to just assume that ‘it’s their maths’. It may well be a mathematical difficulty since problem solving is a real test of whether students have a full understanding of the mathematical concepts underlying the work they are doing. However, it may be that the students, especially if they are second language learners, cannot get to a point where they can even begin to apply mathematical skills. There may be words or phrases in the question that they cannot read. It could be that they can read the words, but do not fully understand what the question means.

1 Item quality

Before helping students to develop strategies for solving written trade calculations, the way the questions themselves are written needs to be looked at. Are they clear and easy to understand or are they full of complicated terms and phrases and unnecessary or confusing information? Could they be rewritten in a better way?

The following points are just some to be aware of when you are writing or selecting written trade calculations.

a. Uncommon or difficult to read words can often be replaced by easier words without changing the meaning of the question.
### Example

<table>
<thead>
<tr>
<th>Example</th>
<th>Possible rewrite</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Determine the missing values in the table relating to spirals...</td>
<td>Work out the missing values in the table about spirals...</td>
</tr>
<tr>
<td>2. Calculate the diameter in mm to turn the end prior to milling the square.</td>
<td>Work out the diameter in mm to turn the end before milling the square.</td>
</tr>
<tr>
<td>3. Two metres in length...</td>
<td>Two metres long...</td>
</tr>
<tr>
<td>4. A plumber is required to construct...</td>
<td>A plumber has to build...</td>
</tr>
<tr>
<td>5. What is the force required to maintain motion of a trailer.</td>
<td>What is the force needed to keep the trailer moving.</td>
</tr>
<tr>
<td>6. What is the distance moved by the clutch throwout bearing...</td>
<td>What distance does the clutch throwout bearing move...</td>
</tr>
<tr>
<td>7. How much gas will be consumed...</td>
<td>How much gas will be used...</td>
</tr>
<tr>
<td>8. How many metres of timber should be purchased...</td>
<td>How many metres of timber should you buy...</td>
</tr>
</tbody>
</table>

b. Difficult or long-winded phrases can make questions unnecessarily complicated. The use of pronouns and the present tense makes questions more personal and easier to read. This also helps students to see themselves actually doing the work, e.g. adjusting the tappets, checking an angle with a sine bar, digging the trench. The changes suggested do not necessarily make the sentences shorter. Sometimes more words are needed to make a point clearly.

### Example

<table>
<thead>
<tr>
<th>Example</th>
<th>Possible rewrite</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. It is specified that 100×75 hardwood bearers be used in a floor 6m×4.8m...</td>
<td>You are laying a floor 6m×4.8m using 100×75 hardwood bearers...</td>
</tr>
<tr>
<td>2. If it should be required to take into consideration a cutting waste of 15%...</td>
<td>If you need to allow a cutting waste of 15%...</td>
</tr>
<tr>
<td>3. Speed variations are obtained with stepped pulleys...</td>
<td>Stepped pulleys vary the speed...</td>
</tr>
<tr>
<td>4. The following components were purchased by a mechanic...</td>
<td>A mechanic bought the following parts...</td>
</tr>
<tr>
<td>5. Calculate the amount of expansion which would occur in a copper pipeline 17m in length if the temperature of the contained water increased from 15°C to 78°C.</td>
<td>How much would a copper pipeline 17m long expand if the temperature of the water in it increased from 15°C to 78°C?</td>
</tr>
</tbody>
</table>
6. A water tank is required to hold 37500 litres and is restricted to a length of 5m and 3.75m in width. What would be the height of the tank?

7. Using sketches as aids determine . . .

8. Determine the angle to set the compound rest to cut the taper illustrated . . .

9. A trade discount of 20% is to be allowed . . .

10. What is the power consumption of a circuit if . . .

A water tank has to hold 37500 litres and can only be 5m long and 3.75m wide. How high would the tank have to be?

Draw sketches and use them to help you work out . . .

You have to cut the taper in the drawing. Work out what angle to set the compound rest at . . .

Allow for a trade discount of 20% . . .

How much power would a circuit use if . . .

Try to write questions as you would ask them in the workshop or out on site. However, if you know that examination questions are generally written in a formal way, you will need to teach your students this kind of language.

c. Many mathematical or technical terms have different meanings in regular English usage.

Examples:
The volume of a cylinder . . .
The volume on a radio . . .
What is the area of a template?
Which area do you live in?
The circular pitch of a spur gear . . .
The cricket pitch . . .
The engine developed power.
You get a film developed.

d. There are many different ways of asking the same type of question. You need to make sure your students understand this varying phraseology.

Examples:
Area calculations can be written as:
• find the area of . . .
• what is the surface area in m$^2$ of . . .
• calculate the total friction area of . . .
• find the cost of painting the outside of . . .
• what is the cross-sectional area of . . .
• find the road surface contact area of . . .
• calculate the total opening in an engine cylinder which has . . .
• how many square metres of covering . . .
• how much paint would you need to cover . . .
• estimate the number of m$^2$ of tiling needed . . .
• what is the total bolt area under stress . . .
• what is the catchment area of . . .
Volume calculations can be written as:

- calculate the cubic capacity of an engine...
- calculate the engine displacement in cc and litres of a...
- calculate the swept volume of...
- find the volume of a footing...
- how many cubic metres of concrete will you need...
- calculate the quantity of excavated material...
- estimate the cubic contents of...
- calculate the amount of filling required...
- how many litres of water will a tank hold...
- how many m$^3$ of soil must be carted away...

e. The question may contain unnecessary information or may lack certain details needed for a solution. Teachers often assume trade knowledge which their students do not have. As a check always proof-read your own questions and ask someone else, preferably from outside your trade, to read them through for you. They will often pick up ambiguities or other problems in question format that you have not noticed.
Chapter 7: Problem solving

Problem solving involves a lot of decision making. You have to decide what the question means, what information you need and what you don’t need, what to do first, which operation to use and whether your final answer is reasonable. To further complicate the process you may make a careless computational error somewhere along the way or worse still press the wrong button on your calculator.

For many students, problem solving is something they feel they can’t do. As teachers you need to build up student confidence about problems and convince them that not knowing doesn’t mean failing, it just means not having found out yet. You can teach strategies and ways of attacking a problem, which can help students to develop their own problem solving abilities. By exposing students to a range of different problems and focusing on the process of obtaining the solution (not just the answer itself), you can help your students to develop the skills and flexibility needed for the practical demands of the work situation where no two ‘problems’ are ever exactly the same.

Properly used, the calculator can be a confidence builder and can increase the problem solving efficiency of its user. Calculators do not solve problems, people do; but they can free students from long and cumbersome calculations and allow them to concentrate on the more important aspects of the problem solving process.

There are four basic steps in the problem solving process.

- Get to know the problem.
- Choose what to do.
- Do it.
- Look back.

Teaching students these steps gives them a structure to use to get started in the problem solving process. Students will need a lot of assistance and examples to work through before they become confident problem solvers. It would be useful to include a unit of work aimed at developing problem solving skills in the curriculum for the early weeks of Stage 1 trade courses. As trade teachers you have many excellent opportunities for teaching ‘real’ problem solving in practical situations.
Get to know the problem

Reading for understanding

Reading mathematical material is a different task to reading narrative prose. When you read a newspaper or a novel you tend to read fairly quickly with only slight attention to detail. You are concerned with the overall meaning of the passage. In comparison, written mathematics problems contain a lot of information in a few words and often include technical vocabulary and symbols.

You may need to read tables, charts and graphs as well as interpret ratios, scales, formulas and drawings. You should read such material slowly, carefully and critically and be prepared to re-read it several times to make sure that you really understand what you have to find out.

The best way to check students’ understanding of the concepts and vocabulary in a question is to get them to rewrite the problem in their own words. Let them discuss the meaning of the problems in pairs or groups; by using and rephrasing the mathematical language, they will gradually come to understand it.

Teachers could also:

- Let the students work through a set of problems writing down what information they know and what extra information they need to know to work out a solution.
- Give students a page of written calculations and let them work through them noting which numbers or information they do not need to use. Students often think they must use all the digits mentioned in a problem. This sorting of information is an important aspect of problem solving as real life problems always contain extra information.

For example, give students problems like the following:

A vehicle uses 56 litres of fuel on the forward journey and 48 litres on the return journey. If the single journey was 680 km and the fuel cost 54.7 cents per litre, what was the total cost of fuel for the complete trip?

In this example the distance is unnecessary information.

Work out the stress in a drive belt with dimensions 50 × 5 × 300 mm. The force applied to the belt is 900 N.

Here you don’t need to know the thickness of the belt to work out the answer.

Using a diagram

Drawing a diagram or rough sketch can often help students to imagine the practical situation and increase their understanding of the problem. The diagram should show the relative sizes involved and any other important information.

A 12 volt ignition circuit has a 1.5 ohm resistor connected in series with the coil. The current flow in the circuit is 3 amperes. Calculate the resistance of the coil.
At first reading, this question could be confusing to students who are used to using the formula \( R = \frac{E}{I} \) because the question gives a value for all three bits of the formula. However, drawing and labelling the diagram helps the student to see what has to be done.

\[ \begin{array}{c}
3A \\
\downarrow \\
1.5\Omega \\
\downarrow \\
r = ? \\
\end{array} \]

The resistance given is for only one resistor. It is the resistance in the coil that is needed.

**Choose what to do**

Choosing a method or strategy to solve the problem is perhaps the most difficult part of the problem solving process. Teachers need to encourage students to focus on the thinking involved, not just getting the answer. Emphasise persistence, not speed in tackling problems. Let students discuss in pairs or groups different ways of solving the problem, which operations they will need to use and the different steps they will need to work through before reaching a solution. Encourage them to think about when they have come across a problem like this before and remember what they did then. Help them to look for connections between the information they have and the results they want. The question really is ‘How do I get from what I know to what I need to know?’

One strategy is to suggest to students that they make an easier problem by replacing the original hard numbers with easy ones, and imagining they are doing the activities themselves.

*e.g.* A car travels 195 kilometres and takes 2 hours 15 minutes to reach its destination. Find the average speed in km/h.

Say to the student:

If you drove 200km and it took you two hours, how fast were you going?

Most students can do this problem and they will say ‘100km/h’. The answer makes sense to them. The next step is to ask, ‘How did you get 100km/h?’ If they don’t know you may need to repeat the process with other easy numbers until they realise that they divided the 200 by the 2, i.e. divided the distance by the time.

Students also have to decide:

- which formula to use, e.g. \( \pi r^2 \) or \( \frac{\pi d^2}{4} \).
• when to convert metric units, e.g. mm to m.
• which parts of the calculation to do first.
• how to lay out the problem.
• what a reasonable answer to the calculation might be.

For example:

A six cylinder engine has a bore of 86mm and stroke of 75mm. Work out the engine displacement.

To choose what to do, the student has to:

(a) know that engine displacement is equivalent to the volume of all the cylinders;
(b) remember the formula \( V = \frac{\pi d^2 h}{4} \);
(c) know that the bore is the diameter \( d \) and the stroke is the height \( h \);
(d) remember to multiply the volume of one cylinder by 6 to get the engine displacement;
(e) realise that it will be easier to change mm to cm before substituting in the formula so that you get the answer in cc's;
(f) divide the cc answer by 1000 to get the answer in litres.

Asking students to list all the steps involved, as in the above example, without working out the problem, gives them practice in this aspect of problem solving.

Do it

Carrying out what you have decided to do is often a routine, computational process. The calculator is a good tool at this point. Students should be encouraged to estimate the results of their calculations before using the calculator: see Chapter 3. If teachers expect students to make estimates, they should always do them themselves when working problems on the board or with students individually.

Look back

This is the stage of problem solving that is most often neglected. When students arrive at an answer, they often think their job is finished. Even if they have made an estimate, there is no guarantee that they will actually go back and compare it with their answer. It is, however, the step that takes them closest to the adult attitude of aiming for 100% accuracy. It means that the problem solver is not satisfied with any answer. He or she wants the right answer.

Suggesting that they always write out an answer using the words of the question will help make sure that they look back. They also need to:

• go back and check that they have answered all parts of the question;
• compare their answer with the estimate;
• ask whether the answer is reasonable.

This last point is particularly important. If students are giving ridiculous answers such as 250 cubic metres of concrete to pour a small garage floor, it may
be because they simply haven’t asked themselves whether it really could be that big. They need to be encouraged to listen to their brain saying ‘that can’t be right!’ and work through the problem again. It helps if they can work with another student to discuss their solution. Often explaining what you did to someone else helps you to see your mistakes.

Worksheet 7.1 is a suggestion for giving students practice in checking the reasonableness of answers. The idea is that students do no working out. They should just think about the alternative answers and decide which one is most likely to be right. It would be even better to make up a similar trade specific exercise. As with all the other stages of the problem solving process, teachers should always discuss the reasonableness of the answer with the students when they are working a problem on the board.

It is important that problems always use realistic figures such as up to date prices and sensible measurements so that students can use their knowledge of the real world when checking their answers. The strategies to teach concepts of size talked about in Chapter 3 will make sure that students can imagine what their answers mean. They will be able to answer questions such as, ‘Would a bolt be that small?’ ‘Would you need that much concrete to lay a path?’ If they know the average sizes and recent prices they will be better able to make these judgments.

Conclusion

The key to successful problem solving lies in thinking about what you are doing at every step. Even if answering a question that does not involve any real decision about choosing what to do (Step 2), because the teacher or the trade practice dictates the method, all the comments in this chapter about the other steps still apply. In fact, even when doing a simple sum such as $0.5 + 0.7$, students should be encouraged to think about what it means and so be alert to what is a reasonable answer. In this case they might think, ‘$\frac{1}{2}$ plus more than $\frac{1}{2}$ means the answer must be greater than 1’.

A really effective way to train students to think about what they are doing is through discussion. The more talk there is about calculations the better. Question students continually:

- What does this question really mean?
- Why do we use this method?
- What do you think the answer will be, approximately?
- Is there any other way to do this?
- Could this answer be right?
- Does this remind you of another part of our course?

Whenever students make errors in written work, instead of simply marking them wrong, teachers should lead them to reason out the right answer. Marking their work with a series of ticks and crosses gives few clues to what the right processes are. If there isn’t time to talk to all students individually, make plenty
of written comments on their attempted solutions. Hold back from always giving them the correct answer too soon, and suggest clues that will help them to work it out for themselves, so that they learn from their mistakes. Students should be taught to keep working at a problem until they get it right.

It is the problem solving process that has to be learnt. Questioning and analysing errors are ways to achieve good problem solving skills.
Worksheet 7.1
Checking the Reasonableness of Results

Which one of these answers is most likely to be correct?

1. My car's petrol tank holds: 15L 50L 200L

2. The amount of concrete needed for the garage floor will be: 2m³ 0.2m³ 12m³

3. The new-born baby weighs: 7.3kg 3.7kg 0.735kg

4. The area of this circle is: 94mm² 150mm² 707mm²

5. The height of the prison escapee is: 140cm 170cm 240cm

Answers
1. 50L 4. 707m²
2. 2m³ 5. 170cm
3. 3.7kg

Appendix: Worked examples

The following are questions taken from exam papers or textbooks. On the right is what might be written on the board or in a student’s book. On the left are notes on points that might be made at each step of the problem solving process. It is not intended that these points would all be made by the teacher, but rather that they would come out through questioning and discussion with students. It is also expected that the technical explanations would be made at the same time as the mathematical explanations, using either models or the real thing wherever possible.
Worked example 1: notes on the problem solving steps

Get to know the problem
You have to make a taper that has a diameter 29mm at one end and 11mm at the other. The piece is 74mm long but 12mm of this is flat. Therefore, the tapered length will be 74 - 12 = 62mm.

The angle you have to work out is the angle formed by the side of the taper and a horizontal line. The best way to work out an angle is to find a right-angled triangle that includes the angle you want, so that you can then use trigonometry.

Looking at the diagram, imagine that the 11mm wide flat section on the right extends to the left through the middle of the taper. Now imagine that you remove this section, leaving one large triangle. Draw this simpler diagram.

You can see that this has not changed the slope of the sides. The height of this triangle will be the large diameter minus the small diameter, which you have just removed (29 - 11 = 18mm). The angle formed where the two sides meet (called the included angle) is twice the angle you need to cut the taper. If you draw a line which cuts this angle in half, you will get two identical right-angled triangles. Each triangle contains the angle you want, which we will call 'a'. Add this to the diagram.

Choose what to do
If we look at either of the two right-angled triangles, we already know the length of the base (62mm). This is the side adjacent to the angle we want to know. We can also work out the opposite side. It is 18 + 2 = 9. So, now we know the opposite and adjacent sides to the angle we want to know. The trig ratio that uses the opposite and adjacent sides is the tan ratio.

Do it
Make an estimate first.

Look back
The answer is close to the estimate. It is a small angle and it looks right for the diagram. It matches what you usually use in the workshop.
Worked example 1: fitting and machining

You have to cut the taper shown in the drawing below. Work out what angle you would set the compound slide at.

\[ \tan a = \frac{\text{Opposite}}{\text{Adjacent}} \]
\[ = \frac{9}{62} \]
\[ = 0.14516129 \]
\[ \therefore a = 8.26^\circ \]
\[ = 8^\circ15' \]

Estimate:
\[ \frac{6}{60} = 0.1 \]
This will be a small angle.

You would set the compound slide at 8°15' to cut the taper.
Worked example 2: notes on the problem solving steps

Get to know the problem
First, draw a plan of the roof. Remember the roof has two sides and a rise. The roofing sheets will be the length of the common rafter. The rise, half span and the common rafter form a right angled triangle. The common rafter is the hypotenuse. Draw a second diagram to show this. If we work out the length of the common rafter, we will know the length of the roofing sheets. To make an order we will also have to work out how many sheets are needed.

(a) Length of common rafter

Choose what to do
As we know the length of two sides of a right angled triangle, we can use Pythagoras' Theorem to work out the length of the third.

Do it
We can use our knowledge of the relative length of the hypotenuse and the longer of the other two sides of the triangle to get an estimate before calculating the answer.

Look back
This answer fits with our estimate.

(b) Number of sheets needed

Choose what to do
The effective cover is the width of one sheet, minus the overlap. We know the length of the ridge, so if we divide the length by 760, we will have the number of sheets needed for one side.

Do it
Estimate the answer before doing the division. The exact number is 17.76. However, you can't buy 0.76 of a sheet so you will have to buy 18 for each side. If the exact number had been 17.36 you may have been able to cut a sheet into two pieces. Then you could have bought 17 for each side plus one to split, 35 altogether.

Look back
The answer is close to the estimate. It sounds right.
Worked example 2: plumbing or carpentry and joinery

Calculate: (a) the length of roofing sheets; and
(b) the number of sheets needed, to cover a gable roof.

The dimensions of the roof are: ridge length 13500, span 7500, rise 1490. Effective cover of the sheets is 760. (All measurements are in millimetres.)

\[ CR^2 = \text{rise}^2 + (\frac{1}{2} \text{ span})^2 \]
\[ = 1490^2 + 3750^2 \]
\[ = 16282600 \]
\[ CR = \sqrt{16282600} \]
\[ = 4035 \]

Estimate: The hypotenuse will be not much longer than 3750, probably around 4000. It can’t be any more than about 5000.

The length of the common rafter and therefore the roofing sheets is 4035mm.

No. of sheets for 1 side \[= \frac{13500}{760} = 17.76 = 18 \]

Estimate: \[\frac{14000}{700} = 20\]
Total needed will be about 40

No. of sheets for 2 sides \[= 18 \times 2 = 36 \]

The roof will take 36 sheets.
Worked example 3: notes on the problem solving steps

Get to know the problem

The idea of a brake is that a relatively small force (effort) on the end of the lever creates a much greater force (load) in the push rod that will eventually create a force large enough to stop the car. The pivot at the top acts as the fulcrum.

Draw a simpler diagram, showing the distances and the forces.

Choose what to do

We know two distances from the fulcrum and one force. It is a moment's problem. The moments have to be equal.

Do it

Make an estimate first.

Look back

The answer is close to our estimate. The force exerted at the push rod is much greater than the original force. This makes sense.
Worked example 3: automotive engineering

A 260mm long pendant type brake pedal is pivoted at the top end. The master cylinder push rod joins 50mm from the pivot. If the driver exerts a force of 120N on the pedal, what force will be exerted at the push rod?

\[ \text{Load} \times 50 = \text{Effort} \times 260 \]
\[ = 120 \times 260 \]
\[ \text{Load} = \frac{120 \times 260}{50} \]
\[ = 624 \]

The force exerted at the push rod is 624N.
Worked example 4: notes on the problem solving steps

Get to know the problem
Establish at the beginning that the final gear will be slower than the driver because the driver turns many times to make the end gear turn once. In fact, the final gear will be much slower than it would be if there were only the driver and the final gear in a simple gear chain because the effect of the middle gears is to slow it down.

In this problem, one turn of the driver 1 will mean its teeth mesh with only 32 of the follower 1's 64 teeth (i.e. follower 1 only turns half way round). Driver 2 is fixed to follower 1 so it only turns half way round too. A half turn will mean only 12 of its teeth will mesh with the 72 teeth of the final gear, i.e. the final gear will not move much at all. If you only had two gears (driver 1 and another follower) the follower gear would have to be very much bigger than the follower in this problem to turn as slowly.

If we were to work out exactly how much bigger than the driver the final gear would have to be in a simple gear train to turn this slowly, we would have the overall gear ratio. We could then use the overall gear ratio to work out how fast the final gear turns. If it acts as if it were 10 times as big, it would turn at one tenth the speed.

(a) Overall gear ratio:

Choose what to do
Driver 1 and Follower 1 are a pair; Driver 2 and Follower 2 are a pair. The first simple gear ratio is \( \frac{1}{2} \). The second is \( \frac{1}{3} \). So the overall gear ratio would have to be \( \frac{1}{2} \) of \( \frac{1}{3} \). The two simple ratios will have to be multiplied.

Look back
An overall gear ratio of \( \frac{1}{6} \) means that the final gear acts as if it were six times bigger than the driver in a simple gear train. This sounds right. The point of having a compound gear train is that you don’t have to use such huge gears.

(b) Speed of the final gear:

Choose what to do
If the final gear acts as if it were six times as big it must turn at \( \frac{1}{6} \) the speed.

Look back
This is much slower than the driver, as expected.
Worked example 4: fitting and machining

In the compound gear train shown in the diagram find (a) the overall gear ratio and (b) the speed of the final gear.

**Driver 1** - 32 teeth
**Follower 1** - 64 teeth
**Driver 2** - 24 teeth
**Follower 2** - 72 teeth

**Speed of Driver 1 = 420rpm**

(a) **Overall gear ratio = simple gear ratios multiplied together**

\[
\text{Simple gear ratio} = \frac{\text{no. of teeth in driver}}{\text{no. teeth in follower}}
\]

\[
\text{Gear ratio 1} = \frac{32}{64} = \frac{1}{2}
\]

\[
\text{Gear ratio 2} = \frac{24}{72} = \frac{1}{3}
\]

\[
\text{Overall gear ratio} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}
\]

(b) **Speed of final gear = Speed of driver 1 \times overall gear ratio**

\[
= 420 \text{rpm} \times \frac{1}{6} = 70 \text{rpm}
\]

The final gear turns at 70rpm.
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TRADE MATHEMATICS: A HANDBOOK FOR TEACHERS is a resource book for teachers on mathematics topics common to many trades. The main approach is that problem-solving strategies should be applied in all trade calculations. There are strong arguments, backed up with practical suggestions, for teaching for understanding to make trade mathematics as straightforward as possible.

Topics covered include: decimals
ratios, fractions and percentages
estimation
teaching concepts
formulas
written calculations
problem solving

The handbook deals with major difficulties for students in these topics, with teaching ideas on how to overcome them. Several chapters include sample worksheets which can be copied for class use, and the book concludes with trade examples worked in line with the key principles.
The authors are employed by the NSW Department of TAFE. Jeannette Thiering is Head of Division of Adult Basic Education, Janice McLeod is a Literacy and Numeracy teacher at Wollongong College of TAFE and Sue Hatherly is a Senior Education Officer in the Staff Development Division.

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