TAFE National Centre
for Research
and Development

MATHEMATICS IN TRADE COURSES

ROSEMARY OSMAN
TAFE National Centre
for Research and Development

MATHEMATICS IN TRADE COURSES

Rosemary Osman

Adelaide 1984
The early work undertaken by Rosemary Osman in the area of mathematics for trade students gave rise to a paper entitled *Trade mathematics matters: Draft report*. That paper, and the work devoted to its preparation, were used as the basis for a national workshop/seminar held in Adelaide in August, 1982.

Subsequently, a project on mathematics in trade courses has been commenced, involving the South Australian College of Advanced Education, the SA Department of Technical and Further Education, and the TAFE National Centre for Research and Development.

The materials from this project may not be available for some time. Meanwhile, it was felt that the draft report should be made more readily available as a working paper. In order to facilitate work on the mathematics project, comments on this working paper are welcomed, and can be forwarded to Ms Di Skott.

I would like to thank Rosemary Osman for her valuable contribution to this topic, the members of the discussion group (listed in Chapter 1), Annabel George and Giulia Reveruzzi for word processing assistance, and Di Skott for her valuable assistance throughout the project.

In common with all working papers, the views expressed are those of the author and do not necessarily reflect the views of the Board or staff of the TAFE National Centre for Research and Development.

Graham D. Hermann
Co-ordinator
## CONTENTS

<table>
<thead>
<tr>
<th>FOREWORD</th>
<th>iii</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHAPTER 1:</td>
<td>INTRODUCTION</td>
</tr>
<tr>
<td>1.1</td>
<td>Getting started</td>
</tr>
<tr>
<td>1.2</td>
<td>Discussion of perceived problems</td>
</tr>
<tr>
<td>1.3</td>
<td>The issues</td>
</tr>
<tr>
<td>CHAPTER 2:</td>
<td>RESEARCH ON LEARNING AND TEACHING MATHEMATICS</td>
</tr>
<tr>
<td>2.1</td>
<td>The difficulty of learning mathematics</td>
</tr>
<tr>
<td>2.2</td>
<td>Teaching methods</td>
</tr>
<tr>
<td>2.3</td>
<td>Background psychology</td>
</tr>
<tr>
<td>2.4</td>
<td>Content of the curriculum</td>
</tr>
<tr>
<td>CHAPTER 3:</td>
<td>MATHEMATICS COUNTS: SOME FINDINGS WITH IMPLICATIONS FOR TAFE IN AUSTRALIA</td>
</tr>
<tr>
<td>CHAPTER 4:</td>
<td>A RESPONSIVE PROCESS FOR FACILITATING CHANGE</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>4.2</td>
<td>A brief review of research on change</td>
</tr>
<tr>
<td>4.3</td>
<td>The process of change</td>
</tr>
<tr>
<td>4.4</td>
<td>Diagnostic tools</td>
</tr>
<tr>
<td>4.5</td>
<td>Supporting classroom change</td>
</tr>
<tr>
<td>4.6</td>
<td>Staff development</td>
</tr>
<tr>
<td>4.7</td>
<td>Networking and dissemination</td>
</tr>
<tr>
<td>4.8</td>
<td>Implications for TAFE Authorities and mathematics in trade courses</td>
</tr>
<tr>
<td>CHAPTER 5:</td>
<td>ONE POSSIBLE APPROACH TO TEACHING MATHEMATICS IN TRADE COURSES</td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>5.2</td>
<td>Teaching for conceptual understanding—guidelines</td>
</tr>
<tr>
<td>5.3</td>
<td>An example: hydraulic brake systems</td>
</tr>
<tr>
<td>5.4</td>
<td>An example: area</td>
</tr>
<tr>
<td>5.5</td>
<td>An example: plumbing—an introduction to pipe sizing</td>
</tr>
<tr>
<td>5.6</td>
<td>An example: volume</td>
</tr>
<tr>
<td>CHAPTER 6:</td>
<td>SUMMARY</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>55</td>
</tr>
</tbody>
</table>
CHAPTER 1
INTRODUCTION

1.1 GETTING STARTED

An initial literature review of the nature of the possible perceived problems in mathematics in trade courses focused attention on:

- the appropriateness of the mathematical content of trade curricula
- current teaching methods and appropriate resources
- relationships between TAFE, secondary schools, mathematics educators and industry.

It seemed most likely that a better understanding of the relevant issues would be gained by facilitating discussions between trade teachers and secondary mathematics consultants, mathematics education lecturers, and secondary mathematics teachers. Improved understanding could then open up possible strategies for overcoming problems associated with mathematics in trade courses.

1.2 DISCUSSION OF PERCEIVED PROBLEMS

It was decided to set up an initial discussion group in Adelaide (for administrative ease). To facilitate a meaningful discussion, the size of the group was kept relatively small.

The group included TAFE lecturers with various trade backgrounds, some of whom were also student counsellors, mathematics educators from the S.A. Education Department and the S.A. C.A.E., and Bev Beasley and Rosemary Osman from the TAFE National Centre. Alan Bell, from the Shell Centre for Mathematical Education, Nottingham University, also attended the first meeting, and I would like to acknowledge his contribution.

Membership of the discussion group

<table>
<thead>
<tr>
<th>Name</th>
<th>Institution</th>
<th>Background</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jeff Baxter</td>
<td>SACAE—Sturt Campus</td>
<td>Mathematics Educator</td>
</tr>
<tr>
<td>Bev Beasley</td>
<td>TAFE National Centre</td>
<td>Project Officer</td>
</tr>
<tr>
<td>Laurie Carter</td>
<td>Panorama Community College</td>
<td>Student Counsellor, Fitting and Machining</td>
</tr>
<tr>
<td>David Fewster</td>
<td>Regency Park Community College</td>
<td>Plumbing Lecturer</td>
</tr>
</tbody>
</table>
During the first meeting, Alan Bell acted as an 'outside expert'. He has credibility, impartiality, and provided an authoritative voice on mathematics education research and teaching methodology. He provided major input on the results of recent mathematics education research, and summed up the issues the group had discussed.

Reports of the discussions were collated in the form of issues that arose. We concentrated on those issues which were repeated in some form, that is, those which came up often in discussion, and those which engaged us in prolonged talking and were obviously perceived as significant.

Two meetings of this discussion group were held, and the major issues arising during the discussions are described below.

How are students being helped now?

The kinds of assistance that students in difficulties are receiving ranges from simply being acknowledged as experiencing difficulty to receiving sustained assistance both within and outside the trade classroom. In some cases, students with problems are identified through testing, which is mostly the testing of discrete mathematics skills.

In some colleges, pre-testing is done before the apprenticeship is begun or very early in the apprenticeship. Sometimes individual counselling follows the test. The results of tests may be given to students and teachers or they may be confidential and the students' performances indicated in other ways. Some students are offered withdrawal classes in their own, or in class time, and attendance may be compulsory or voluntary. Occasionally, students are streamed into different performance groups. In other cases students are identified, formally or informally, but receive no special assistance either in the classroom or out of it. Sometimes there is also post-testing which may be intended to check the reliability of the first test or to check the effectiveness of any existing remediation program.
The following issues were collated in the form of questions, because that is how they arose in conversation and because so many of them remained unanswered during the discussion.

- How reliable are the tests of discrete skills?
- Does the language of such tests make even understanding the questions difficult?
- In these tests are students able to make use of what they already know and demonstrate what they do know?
- Are these tests related to the real purposes of mathematics—which is to apply knowledge to particular problems?
- To what extent are students undertaking these tests affected by largely uncontrollable variables, for example, the apprehension caused by doing tests?
- Do tests, no matter how sensitively administered, destroy the confidence of students who find themselves overtly identified as 'having problems'?
- Can these students be identified by the TAFE teacher as effectively or more effectively than by testing?
- When these students are identified, by whatever means, do the TAFE teachers then know ways of helping them?
- Does the remedial teacher have ways of helping them with their trade classroom work?
- Can post-tests reliably pick up causal links between progress (or regression) and what has happened between one test and another? Can we, with any accuracy, say our remediation program has caused this improvement?
- Is it possible that test results may show improvement when the student has shown little improvement in the classroom, that is, has the student merely got better at doing tests?
- Do the tests which are used assess skills relevant to apprentices?

When should remediation occur?

The following points were made during the discussions:

- When students first start their courses in TAFE colleges, little is known about the nature of their specific problems. So if remediation occurs at the beginning of the year, it has to be done in a general way because the students haven't had the benefit of the trade course. It's also no good giving a remedial course in mathematics that's supposedly going to last students for three or five years if they're not going to be using the mathematics learned, because they will forget most of it.
Learning occurs effectively and efficiently when it is clearly related to the student's previous experience and present needs.

Remediation needs to occur on the spot, where it's directly relevant to the specific problem. The most effective way of teaching mathematics relevant to a specific problem is to teach the mathematics at the time the problem arises.

TAFE does not have the resources to provide a remedial teacher as support for every TAFE teacher. Most teachers of mathematics in trade courses will have to provide help for their own students.

Teachers of mathematics in trade courses require a 'kit of tools' so that they can help the students with their problems at the time they need it. These 'tools' are not available at the moment. The teachers will need to have the confidence in their own ability to identify the nature of a problem, and solve it.

**Applying mathematics knowledge in problem solving**

In the discussions, it was suggested that mathematics taught in the trade classroom may, in some cases, be very removed from any real context, and any apprentice (including those with difficulties in mathematics) may find it very difficult to transfer discrete, abstract skills learnt at a theoretical level to practical situations. Essentially the apprentice needs to be able to make use of mathematics on the job. This is the main purpose of mathematics teaching in trade courses. Given this requirement, many aspects of existing teaching practices need to be examined. The following concerns are not listed in any order of priority.

In every work situation there are many different problems to be solved. Is it known what the mathematics requirements are for each of these problems in the real life industrial setting? In other words, is it possible that trade teachers are teaching mathematics which is no longer relevant? Or that trade teachers are failing to teach areas of mathematics which are being used?

How much use is being made of calculators in on-site work?

How long is it since the trade teacher worked in the particular industrial area? How much opportunity does he or she have to observe/participate in on-site work situations?

How related to the real work situation is the language of the materials being used in the trade classroom?

While a great deal of applied mathematics in trade problem solving is essentially lock-step (i.e. one procedure must be completed before the next step can be carried out), the procedures for working out how to do the mathematics for any section may vary considerably.

What are the various ways the tradespersons on-the-job handle their 'applied mathematics'?
Trade mathematics and the primary/secondary school

The following points arose in the discussions:

- Certain areas of mathematics are no longer taught at primary or secondary level but some trades still make use of them, for example, the Imperial system and the use of logarithm tables may not be taught in all schools. The question may not be whether the schools should re-introduce them, but rather that the trade concerned should try to use procedures which are being taught at primary/secondary levels, for example, the use of conversion tables and calculators.

- Mathematics in some trades often draws on what is learned at the primary level, which apprentices appear frequently to have forgotten.

- While it may not be the job of the secondary mathematics teacher to give mathematics teaching for specific trades (nor is it possible given the great diversity of mathematics requirements across the trades), it may be that a stronger emphasis on teaching mathematics to solve 'real' problems would be very useful for apprentices-to-be.

Obviously closer communication between school mathematics teachers, trade teachers and curriculum writers from both areas would benefit all concerned. However, knowing about mathematics education at the primary/secondary level hardly assists the trade teacher with those students who have profound mathematics difficulties and who are in the classroom now.

Need for dialogue between teachers of mathematics in trade courses and mathematics educators

Points which arose in the discussions include:

- In teaching the concept of division, for example, we often concentrate on the symbols, or teach an algorithm, rather than explain what 'division' means. Too often people try to remediate the 'representation' of a concept (e.g. symbols), or the rules associated with a 'concept' (e.g. an algorithm), rather than the concept itself (i.e. division in this example). Remediating the representation is usually short-term in its impact.

- The impact of a well-chosen methodology is overwhelming. We're not talking about merely understanding or not, but knowing or not knowing.

- We need to set up a mechanism for dialogue and exchange of ideas between trade teachers and mathematics educators. We need to get the mathematics educators to explain the methodologies they have access to, and the trade teachers to explain the concepts and skills that they are having difficulty teaching.
Need for reviews of the mathematical requirements of trade courses and methodology assistance for teachers of mathematics in trade courses

The following points were raised in the discussions:

1. Ultimately, the trade teacher is responsible for teaching mathematics to his apprentices.

2. Some trade teachers recognise that a student is having problems, but don't know what to do. These teachers need some assistance in identifying the nature of the problems that the student has, and they need some techniques for helping the student through these problems.

3. Within a class there is quite a range of student performance. Teachers may need assistance in teaching techniques to handle these 'mixed performance' groups. For example, the use of a roofing square could be taught to the whole class, but an advanced roofing module could be offered for students who elect to learn about the geometrical developments behind the use of the roofing square.

4. Teachers and curriculum writers may also need to have a critical look at the necessity of the mathematical content of their courses. The trade student doesn't need the skills of a technician, but needs some very basic mathematical skills and processes to be able to survive as a tradesperson. For example, when someone in the workshop is going to drill a hole, does he really sit down and transpose a formula to obtain the drill speed? Or does he refer to a chart of drill speeds? Or does he ask someone? Perhaps we should be teaching students to read charts and/or ask questions rather than transpose formulae.

5. We should be concerned also about assisting our students so that, in any situation, they can be independent problem solvers. People's modes of learning and of applying their learning vary considerably. Are we teaching our students to become capable of rehearsing problem solving by imagining how they would tackle any problem? What would I do if . . .? The lowest level of performance is not enough.

6. We need a starting point for those trade teachers who are interested in the problems associated with mathematics in trade courses and for raising the level of awareness of trade teachers who are not conscious of the problem or who are conscious of the problem but not of the urgent need to tackle it.

1.3 THE ISSUES

It is possible to pick out several themes underlying the issues discussed in the previous section.

1. The first of these is the need to be aware of recent research into teaching and learning mathematics and the implications for TAFE.
During the last six years there has been a growing worldwide interest in mathematics education research. The Mathematical Education Research Group of Australia (MERGA) held its first Conference in 1977. Until then, there was no national professional group in Australia directly concerned with research into teaching and learning mathematics.

The Australian Mathematics Education Project (AMEP), a Curriculum Development Centre project, was only established in 1980. The establishment of MERGA and AMEP have both stimulated research and the dissemination of research results.

Dr. Alan Bell, from the Shell Centre for Mathematical Education, Nottingham University, has recently completed a review of the research on teaching and learning mathematics, as part of the U.K. Cockcroft Inquiry into the teaching of mathematics. A precis of his findings is given in Chapter 2 of this report.

The research into teaching and learning mathematics has major implications for methods of teaching mathematics in trade courses. In particular:

- the research suggests that teaching methods based on conceptual understanding and diagnostic teaching are more effective than methods based on accumulating isolated skills or learning by rote;

- the research stresses the importance of context in learning, and the difficulties associated with transferring skills to new problems.

These research results have major implications for trade teachers, curriculum writers, remedial mathematics teachers, and administrators. This raises several questions of importance to TAFE Authorities, namely:

- what processes exist in the present system to disseminate the results of research and to effect changes in teaching practice
- how effective are the current processes of staff development in TAFE
- does TAFE have people with expertise in mathematics education to act as support and advisory personnel to trade teachers and to initiate research into teaching, learning and mathematics curricula in TAFE trade courses?

2. The second theme underlying the issues relates to the real mathematical requirements of trade students.

How much mathematics does the beginning trade student need to be able to cope with the practical and theoretical components of the trade program?
Essentially the student needs to be able to make use of mathematics on the job. The trade student doesn't need the skills of a technician, but needs some very basic mathematical skills and processes to be able to survive as a tradesperson.

It is difficult to decide what the mathematical requirements of a trade are. There is a temptation to divorce the 'skills' from the problem situation. Once this is done, a new problem is suddenly identified—'students have difficulties in transferring skills'.

Cockroft (1982) investigated the area of employers' needs and the range of mathematics required. Relevant extracts from the Cockroft Report are included in Chapter 3. The implications of their findings for the classroom (paragraphs 83-85, p.24) are particularly relevant for TAFE.

We believe that it is possible to summarise a very large part of the mathematical needs of employment as 'a feeling for measurement' . . . . It implies an understanding of the nature and purposes of measurement, of the many different methods of measurement which are used and of the situations in which each is found; it also implies an ability to interpret measurements expressed in a variety of ways.

3. The third theme relates to the use of tests (screening, achievement, mastery, diagnostic, etc.), the interpretation of student performances on the tests, and the provision of remedial programs.

It would seem that it would be useful to advise TAFE teachers on the evaluation of test materials in terms of facility of use, discrimination, validity, and reliability.

A number of optional strategies and approaches to teaching mixed ability classes could be presented to TAFE teachers to supplement existing ideas and current practices of 'remediation' in withdrawal groups. 'Parallel teaching', for example, might be an effective way of teaching mathematics to students who are having difficulties.

4. The last theme is concerned with the need for liaison between TAFE, State Education Departments, mathematics educators, and industry.

Ideally a school leaver should experience some continuity of learning in mathematics after moving from school to further education and employment. However, this is by no means a simple process. (Cockroft, 1982, para. 162, p.45)

Some TAFE teachers are not aware of either the mathematics curriculum or the teaching methods used in schools, and very few secondary mathematics teachers have ever been near a TAFE College or workshop let alone have any awareness of the applications of the mathematical concepts they teach. Both sectors could only benefit by talking to each other.
One has a feeling that similar communications problems might exist to some extent between TAFE and industry, especially concerning the mathematical requirements on the job.

If some of these 'interface' issues could be solved through dialogue, then perhaps there would be less need for screening tests and remedial programs.

**A responsive process for facilitating change**

An analysis of the perceived problems is not sufficient. We must also develop processes for facilitating change. For example, facilitators could be employed by TAFE Authorities to initiate and co-ordinate developmental programs on teaching and learning mathematics in trade courses. A brief review of recent research findings on the process of change is presented in Chapter 4.

**Emphasis of this paper**

The main body of this paper primarily addresses the first theme raised above, that is, the need for awareness of current research in mathematics education. It does this, however, in a selective way in so far as it is based especially on the Bell et al. and Cockcroft documents referred to in this Chapter.
CHAPTER 2
RESEARCH ON LEARNING AND TEACHING MATHEMATICS

The Cockcroft Committee of Inquiry into the Teaching of Mathematics was set up in 1978 in the United Kingdom following the recommendations of an all-party House of Commons Committee.

As part of this inquiry, a Review of research in mathematical education was prepared by the Shell Centre for Mathematical Education, University of Nottingham. A brief introduction to and summary of this review has been published, including a section on 'Research on learning and teaching' by A. W. Bell, J. Costello and D. E. Kuchemann.

The following chapter is based on extended extracts from Bell's section in the Review of research and the Cockcroft Report (1982).

2.1 THE DIFFICULTY OF LEARNING MATHEMATICS

Many aspects of mathematics look easy because they involve manipulating symbols by following rules or a particular sequence of operations. It is possible to learn how to perform these manipulations without understanding what they mean or the underlying concepts.

For example, the following question was given to a representative sample of fourth year secondary students in the United Kingdom.

Minced beef costs 88.2 cents per kilogram; I buy 0.58kg; to find the cost which calculation must I do:

A 0.58 ÷ 88.2
B 88.2 ÷ 0.58
C 0.58 x 88.2
D 88.2 - 0.58? (Bell et al., 1982, p.3)

This question was answered correctly by 30% of the students.

This illustrates several very important points concerning difficulties students experience in learning mathematics:

- it is important to distinguish between the ability to calculate (with decimals, in this case) and the ability to choose the correct operation
- it is sometimes very hard to choose the correct operation. This cannot be done by a machine, although the calculation can.

Compare the following question with the one above:

Minced beef costs $3.00 per kilogram, I buy 2kg; to find the cost which calculation must I do:
A \[2 \div 3\]
B \[3 \div 2\]
C \[2 \times 3\]
D \[3 \div 2\]

... the choice of operation, not just the calculation, is much easier if the figures are small whole numbers.

Another difficulty in learning mathematics is highlighted by the following problem, concerning the meaning of 'average'.

... consider the now notorious question of Boycott's batting average - 500 runs, 5 times out - How would you explain to a young boy what his average would have been if he had scored 532 runs, and been 5 times out? "You divide the 532 by 5. That gives 106.4 ..." "Yes, but what does his average mean?" "Well, it's what he would have scored in each innings if he had scored the same each time ..." "But how could he have scored 106.4?" (Bell et al., 1982, p.4)

... there is a particular problem of keeping the connection between the symbolic manipulations and their meanings.

2.2 Teaching Methods

The Review of research (cited in Cockcroft, 1982) points out that in teaching mathematics it is possible to distinguish between three elements:

- facts and skills
- conceptual structures
- general strategies and appreciation.

The recognition that these three elements involve distinct aspects of teaching and require separate attention pervades the research on teaching mathematics.

Review of research continues:

**Facts** are items of information which are essentially unconnected or arbitrary. They include notational conventions—for example that 34 means three tens plus four and not four tens plus three—conversion factors such as that '2.54 centimetres equals 1 inch' and the names allotted to particular concepts, for example trigonometrical ratios. The so-called 'number facts', for example \[4 + 6 = 10\], do not fit into this category since they are not unconnected or arbitrary but follow logically from an understanding of the number system.

**Skills** include not only the use of the number facts and the standard computational procedures of arithmetic and algebra,
but also of any well established procedures which it is possible to carry out by the use of a routine. They need not only to be understood and embedded in the conceptual structure but also to be brought up to the level of immediate recall or fluency of performance by regular practice.

**Conceptual structures** are richly inter-connected bodies of knowledge, including the routines required for the exercise of skills. It is these which make up the substance of mathematical knowledge stored in the long term memory. They underpin the performance of skills and their presence is shown by the ability to remedy a memory failure or to adapt a procedure to a new situation. (Cockcroft, 1982, para. 240, p.71)

A combination of experiences, visual perceptions and other information gained by the human senses establish a cognitive record in long term memory. It is this record of information which enables a person with relevant conceptual structures to work from first principles and fundamental understanding when addressing a new situation. Facts and skills can be recalled and applied in informed ways where appropriate conceptual structures exist.

**General strategies** are procedures which guide the choice of which skills to use or what knowledge to draw upon at each stage in the course of solving a problem or carrying out an investigation. They enable a problem to be approached with confidence and with the expectation that a solution will be possible. With them is associated appreciation which involves awareness of the nature of mathematics and attitudes towards it. (Cockcroft, 1982, para. 240, p.71)

If these three elements—skills, concepts and general strategies are distinguished as different kinds of teaching objectives, then appropriate methods should be used for each.

- For strategies and general orientation, a substantial proportion of class time should consist of solving 'real' problems and making mathematical investigations.

- For conceptual learning, diagnosis of students' existing concepts and the provision of problems based on familiar situations and concrete materials, with conflicts of understanding resolved by discussion.

- For essential facts and skills, first establishing understanding and inter-connections with the rest of the [student's] knowledge, then drill and practice to increase fluency. (Bell et al., 1982, p.8)

The Review of research concludes that the orientation given to the work by the teacher has been shown to govern the kind of knowledge which the pupil builds up, determining, for example, whether it is superficial and rote-memorised, or understood and deep.
Other research results concerning teaching methods look at the question of what constitutes effective, meaningful teaching, in general, and in particular topics.

The Review of research concludes that the most successful methods, and the most plausible theory, use 'cognitive conflict', rather than the accumulation of skills.

In the cognitive conflict method:

i. the teacher presents a problem

ii. the teacher arranges for an apparent contradiction to be met

iii. the learner uses a 'natural' approach

iv. the learner attempts to resolve the apparent contradiction

v. the teacher demonstrates the new approach

vi. the learner performs the new approach, and resolves the apparent contradiction.

Also demonstrated are the importance of immediate feedback of correctness, and the requirements of mastery of one (small) aspect before progressing to another.

In general terms, the above methods might be termed diagnostic teaching. A teacher, using these methods, would monitor the conceptual understanding of the learner by diagnosing the nature of misunderstandings more deeply than normal marking does. Teachers and curriculum writers would also design problems for discussion which enable the conceptual errors to be exposed and corrected by the student.

The Cockcroft Report emphasises that it is not possible or desirable to indicate a definitive style for the teaching of mathematics.

Approaches to the teaching of a particular piece of mathematics need to be related to the topic itself and to the abilities and experience of both teachers and pupils. Because of differences of personality and circumstance, methods which may be extremely successful with one teacher and one group of pupils will not necessarily be suitable for use by another teacher or with a different group of pupils. (Cockcroft, 1982, para. 242, p.71)

Nevertheless, the Cockcroft Committee did believe that there are certain elements which need to be present in successful mathematics teaching to students of all ages:

1. exposition by the teacher

2. discussion between teacher and students and among students themselves

3. appropriate practical work
consolidation and practice of fundamental skills and routines

. problem solving including the application of mathematics to everyday situations

. investigational work. (Cockcroft, 1982, para. 243, p.71)

2.3 BACKGROUND PSYCHOLOGY

Memory

Problem-solving tasks and most learning tasks (such as verbal arithmetic problems, or adding fractions) involve the intake of a number of items of data and the eventual recognition of some connection among them. This involves holding the data in the working memory while scanning it for the required relationships. Research shows that the capacity of this working memory is severely limited (about 3 to 7 items), and this makes it necessary to deal with one small subset of the data at a time, then pass to some other subset—the typical problem-solving approach. This capacity limitation also often leads to misreading of questions, only part of the information being extracted.

The second relevant feature of memory concerns the fact that long term storage and subsequent retrieval depend essentially on the connectedness of the material stored. The simple commitment of an isolated item to memory and its subsequent recall at will almost never occur. What the research shows is:

. that the long term memory stores meanings, and generally not particular forms of words;

. that for memorisation to be effective, strategies such as 'rehearsal', that is, holding the material in consciousness by attending to it, and 'organising' or grouping items by recognising a connection among them, are necessary;

. that if material to be remembered is intrinsically unconnected, some mnemonic device is needed (such as fitting the material deliberately into some scheme prepared for the purpose).

In short, this fundamental research on memory supports and explains the results of the teaching method research in asserting the key importance of meaningfulness and connectedness for successful learning and retention.

The research also shows how deep level memory processing depends on the learner's orientation to understand and consider the material rather than to simply memorise it. This clearly has implications regarding the approach to classroom activity engendered by the kinds of demands made by the teacher, both immediately and in regard to the forms of testing which the pupils expect to undergo.
Effect of context

Mathematical teaching at the moment is usually based on the principles that a given mathematical concept or relationship, such as 'division' or 'proportion', can be learnt in one or more contexts, but abstracted and transferred to other contexts.

For example, one may learn proportion in the context of mixtures for concrete or for metallic alloys, and transfer it to dividing up a pie diagram to show the proportions of time given to work, leisure and sleep during the day.

However, recent research shows that contexts differ greatly in the difficulty they present in the application of the same mathematical structure. Most of the work needed to apply the mathematics idea in a thoroughly familiar context has already been done; this is because relationships existing in the material already exist in the student's mind and have only to be drawn upon when applying the mathematical structure.

For the classroom this implies that:

- the use of a genuinely familiar context may give a first access to the idea for many more students
- abstracting an idea for transfer to another, possibly unfamiliar, context may present difficulties of a quite different order.

2.4 CONTENT OF THE CURRICULUM

The Review of research has shown that there is a very great distance between what can be achieved with understanding by the most and the least able students. It is the teaching method for the least able students which presents the most acute difficulties.

The Review of research shows that:

- if computational skills are not underpinned by conceptual understanding, they are not easily retained; and also that disastrous mistakes occur
- choosing the correct operation can itself be very hard
- meaningful methods are relatively more successful with the less able than the more able.

It is also known that transfer of principles from one context to another is unexpectedly hard. In fact, what to a mathematician looks like the same principle may be a very different procedure in a different context.

Although it is known that the choice of operation, not just the calculation, is much easier if the figures are small whole numbers, numbers can not always be kept simple. The Review of research shows
that, for example, proportion problems in which the ratios consist of
doubling and trebling are quite different, and are done by different
methods, from those with more complex ratios.

Real problems with realistic numbers are needed. This means it is
essential to use a calculator. If the necessary time is spent
teaching students of limited learning capacity 'long' computations,
they will lose the time they need for learning how to apply the
mathematics, which no machine can do for them. The students also need
to be able to round the numbers and check the size of their answer
mentally.

The Review of research concludes that the guidance on the question of
curriculum content given by research is that coherent bodies of
knowledge should be chosen, within which the pupils concerned can
achieve a high degree of mastery. 'The low retention of fragmentary
knowledge is well attested: also clear is the degree of intensity
needed to achieve long term retention and transfer to new situations
which is shown in the successful teaching experiments' (Bell et al.,
1982, p.6).
CHAPTER 3

MATHEMATICS COUNTS: SOME FINDINGS WITH

IMPLICATIONS FOR TAFE IN AUSTRALIA

Given the similarities between the British and the Australian education styles, and the common problems being faced by industry and in trade training, it was felt that many of the findings of the British inquiry could be of relevance to TAFE in Australia.

For this reason some of the findings of the Cockcroft Committee have been summarised. It is left to you to draw particular implications of these extracts for your own situation. I have tried to present sufficient detail to promote greater understanding of the nature, generality, and complexity of the situation.

The full report is available from Her Majesty's Stationery Office in London, or may be ordered through the Australian Government Publishing Service. I would like to acknowledge H.M.S.O.'s co-operation in permitting the TAFE National Centre for Research and Development to print the following extracts.

The numbers in parentheses denote paragraph numbers of the Report.

EMPLOYERS' NEEDS

The Committee found that there was little real dissatisfaction among employers about the standards of mathematics amongst the school leavers they recruit in spite of the alleged volume of complaints that led to the Cockcroft Committee being set up.

'... it has naturally been our concern ... to investigate complaints about low levels of numeracy among young entrants to employment and the need for improved liaison between schools and industry.' (44)

The Committee received some 200 submissions from major companies, industry training boards, the Confederation of British Industry, the Trades Union Congress, the Association of British Chambers of Commerce, and from many small employers. They also studied the reports of research studies into the mathematical needs of various types of employment based on visits to more than 100 companies, and visited 26 firms themselves.

'The overall picture which has emerged is much more encouraging than the earlier complaints had led us to expect. We have found little real dissatisfaction amongst employers with the mathematical capabilities of those whom they recruit from schools except in respect of entrants to the retail trades and to engineering apprenticeships, both of which involve significant numbers of young people.' (46)
'Those who enter the retail trade on leaving school at 16 commonly have very modest or no mathematical qualifications but are often required from the outset to give change, count stock, fill in stock sheets and calculate discounts.' (53)

'The complaints which we have received relating to engineering apprentices seem to stem largely from the performance of applicants in company selection tests, which are very often tests of computational ability only. However, most of the criticism relates to those applicants who are rejected. Employers have expressed comparatively little dissatisfaction with the mathematical performance of those whom they have taken on as apprentices. Moreover, where difficulties, mainly in arithmetic, do arise, we have been told that they can almost always be overcome relatively quickly during the initial apprenticeship year as apprentices gain practical experience in the workshop and so realise why it is necessary to be able to carry out certain specific calculations.' (54)

'It is important at this stage to note that, while agreeing with the importance of arithmetical attainment, the EITB [Engineering Industry Training Board] has stressed to us its view that tests of arithmetical skill play too dominant a role in selection for engineering apprenticeships and that conceptual skills, such as spatial awareness, the understanding of orders of magnitude, approximation and optimisation, are of equal or greater importance.' (55)

Some employers complained of having to undertake 'remedial' training of young employees. Employers should accept that it is necessary to revise skills which may have become rusty through lack of practice, or to introduce applications, possibly of quite elementary mathematics, which may not have been encountered in the classroom, or to teach certain specialised techniques. Employers should not have to 'spend time teaching mathematics which, although it has been included in the school course, has not been understood or is being used incorrectly.' (56)

'In the course of our work we have become aware that a degree of over expectation exists in many quarters as to the level of attainment in mathematics which some school leavers are able to reach . . . This is one of many reasons why good liaison between employers and schools . . . is of such great importance.' (57)

The Committee also highlighted the need for special assistance for the young unemployed: 'lack of use of mathematics on their part may well lead to a degree of "rustiness" which will require sympathetic consideration on the part of employers to whom these young people apply for jobs, and perhaps also the provision of special assistance.' (58)

THE RESEARCH STUDIES

A number of findings emerged from the studies carried out at the University of Bath and the Shell Centre for Mathematical Education at the University of Nottingham.
Both studies paid particular attention to the types of employment open to those leaving school at ages from 16 to 18... More than 90 companies and other establishments were visited [in the Bath study] and a considerable amount of time was spent both in observation of work which was going on and in discussion with employees themselves and with managerial and training staff. The Nottingham study was designed to complement the work undertaken at Bath by looking in greater depth at... specific areas.' (61-63)

The report of the Bath study draws attention to the fact that it is possible when observing work in progress to describe certain aspects of it in mathematical terms. For instance, the mathematical concept of geometrical symmetry is present within many manufacturing processes; it is possible to describe different methods of stacking and packing in geometrical terms or in terms of sorting and classifying. However, even if the mathematical concepts involved have at some time been encountered in the classroom, the employee will probably not consciously analyse in such terms the operations which are being performed; nor, if he were to do so, would he necessarily be able to do his job any better.' (64)

On the other hand, many jobs require the employee to make explicit use of mathematics—for instance, to measure, to calculate dimensions from a drawing, to work out costs and discounts. In these cases the job cannot be carried out without recourse to the necessary mathematics and it is this latter use of mathematics to which we refer in the paragraphs which follow. However, even when mathematics is being used, frequent repetition and increasing familiarity with a task may mean that it may cease to be thought of as mathematics and become an almost automatic part of the job.' (68)

RANGE OF MATHEMATICS REQUIRED

Both studies found that almost all the mathematics which young people need to use, whatever their job, is included within the existing O-level and CSE Mode 1 syllabuses... Since most pupils do not know in advance the type of employment which they will follow in later life, it is important that this should be the case.' (68)

Nevertheless, the studies also identified certain important differences between the ways in which mathematics is used in employment and the ways in which the same mathematics is often encountered in the classroom. We believe that these differences may be among the factors which have contributed to the criticisms which have been voiced in recent years.' (69)

CALCULATION

The need to be able to carry out arithmetical calculations of various kinds is necessary for almost all types of employment investigated.
These calculations are sometimes carried out mentally, sometimes with pencil and paper and sometimes with a calculator. Some jobs specifically require an ability to carry out mental calculations of various kinds. In almost all jobs the ability to carry out some calculations mentally is of value and lack of ability to do this is a frequent cause of complaint by employers." (70)

'Both the Bath and the Nottingham studies found that the methods which are used when at work to carry out calculations with pencil and paper are frequently not those which are traditionally taught in the classroom. Employees use a variety of idiosyncratic and 'back of an envelope' methods, especially for long multiplication and division. Sometimes these methods have been devised by the employees themselves but frequently they have been passed on by their fellow workers. The methods which are used depend very much on the user's confidence in his own mathematical ability. However, at all levels there appears to be a preference for carrying out a calculation in a succession of relatively short stages rather than for making use of a single calculation which is mathematically more sophisticated, and perhaps quicker, but which may be more difficult to use with confidence.' (71)

'The use of percentages is widespread in offices and laboratories, but is much less frequent in workshops. Percentages occur most often in calculations involving money, for example discount, value-added tax, profit or loss; they are also used in a wide range of other calculations in both offices and laboratories. We are aware of a number of complaints from employers about lack of understanding of percentages. These refer not only to clerical staff, whose difficulties can usually be overcome by making use of a formula or a standard calculator procedure, but also to management trainees and even managers.' (72)

USE OF CALCULATORS

'Our own enquiries, and the evidence which we have received, lead us to believe that there is an ambivalent attitude to the use of calculators in industry and commerce at the present time. The Bath and Nottingham studies found their use to be widespread in many types of employment. These include a wide range of clerical and administrative jobs such as accounts departments, banks, insurance offices and related areas of employment. Calculators are also used widely by those concerned with quality and production control; these are all jobs which require a considerable amount of calculation and analysis of data. In all of these situations calculators are regarded as desirable aids to speed and accuracy.' (73)

'Calculators are also used increasingly by many who work on the shop floor but their use is still viewed with suspicion by some managers and supervisors who were themselves trained to use slide rules or logarithm tables. This seems to be especially true of those who supervise engineering and other technical apprentices and craftsmen of various kinds. We are aware of situations in which new apprentices, who had been issued with logarithm tables by their training supervisor, preferred instead to make use of the personal calculators.
which they were encouraged to use on the college courses that they attended. There are, of course, many straightforward calculations which a craftsman needs to be able to carry out mentally but this does not seem to be a reason for denying the use of a calculator when it is sensible and time-saving to use one. However, the majority of young employees who were seen to be using calculators at work had not been trained in their use either at school or on the job. In consequence, calculators were frequently not being used in the most effective way.

(74)

FRACTIONS

'Although fractions are still widely used within engineering and some other craft work these are almost always fractions whose denominators are included in the sequence 2, 4, 8, ..., 64. This sequence is visibly present on rules and other measuring instruments and equivalences are apparent [for example, that 6/16 and 3/8 have the same value]. Addition or subtraction of lengths which involve fractions of this kind can be done directly by making use of the gradations on the rule. When the calculation is carried out with pencil and paper it is never necessary to work out the common denominator which will be required because it is always present already; for example 2 1/4 + 3 5/16 has the necessary denominator, 16, already visible. Even so, the methods used are not always those which might be expected. The report of the Bath study comments that

Frequency of use promotes assimilation of equivalents. This is evident from the following method that a craftsman used to add fractions:

\[
\frac{1}{4} + \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \\
= \frac{1}{4} + \frac{1}{4} \\
= \frac{1}{4} + \frac{1}{4} = \frac{1}{4}
\]

This is another instance of someone rewriting numbers in a more convenient form. It could be described as very mathematical but it is not necessarily the method which would be reinforced in schools.

The need to perform operations such as 2/5 + 3/7 does not normally arise, and manipulation of fractions of the kind which is commonly practised in the classroom is hardly ever carried out. In the rare instances in which it is necessary to multiply or divide fractions, it is usual to convert each to a decimal before performing the operation, if necessary with the help of a calculator.' (75)

'The notation of fractions appears in some clerical and retail jobs, for instance 4 3/7 to represent 4 weeks and 3 days or 2 5/12 to represent 2 dozens and 5 singles. However, school-type manipulation is rarely found and then only in very simple cases; for instance, the calculation required to find the charge for 3 days based on a weekly rate is division by 7 followed by multiplication by 3.' (76)
'One of the more surprising results of the studies is the little explicit use which is made of algebra. Formulae, sometimes using single letters for variables but more often expressed in words or abbreviations, are widely used by technicians, craftsmen, clerical workers and some operatives but all that is usually required is the substitution of numbers in these formulae and perhaps the use of a calculator. The report of the Bath study quotes an example of one such formula which was not even written down, but was remembered by the employee concerned:

A wages clerk explained: "To get the rate of pay per hour, we add together the gross pay, the employer's National Insurance, and holiday stamp money, subtract any bonus, and then divide by the hours worked".

It is not normally necessary to transform a formula; any form which is likely to be required will be available or can be looked up. Nor is it necessary to remove brackets, simplify expressions or solve simultaneous or quadratic equations, although algebra of this kind is sometimes encountered on courses at further education colleges. Solution of linear equations is required very occasionally.' (77)

'Industry and commerce rely extensively on the ability to estimate. Two aspects of this are important. The first is the ability to judge whether the result of a calculation which has been carried out or a measurement which has been taken seems to be reasonable. This enables mistakes to be detected or avoided; examples are the monthly account which is markedly different from its predecessors or the measured dose of medicine which appears unexpectedly large or small. The second is the ability to make subjective judgements about a variety of measures. This is of use in situations in which measurement is difficult or awkward, in which trial and error is possible or in which tolerances are large. Skills of estimation develop on the job but employers often complain that young entrants to industry and commerce lack a 'feel' for both number and measurement, even in terms of the units, whether metric or imperial, which they can be expected to have encountered at school or in everyday life.' (78)

'Although estimation is important, counting and measurement are paramount. A very great deal of the mathematics which is used in employment is concerned with measurement in one or other of a wide variety of forms, by no means all of which are directly concerned with the use of measuring instruments. Measurements are specified in a variety of ways, for instance in terms of number of items or total of money; of length, weight or volume; of ratio, percentage or rate. Use is made of metric and imperial units as well as of units which are peculiar to a particular industry.' (79)
'There are two different aspects of measurement—the first in which an existing measurement such as length, weight or number in stock has to be determined, the second in which it is necessary to create a desired measurement. In every case it is necessary to be aware of how accurate the measurement needs to be; in some cases it is necessary to be able to choose and use an appropriate measuring instrument. The labourer may well measure in terms of buckets or shovels full. The skilled engineering craftsman may work to different tolerances for different parts of a job and will need in each case to choose the appropriate tool or instrument for creating or checking the measurement he requires. The cashier will need to count the money in the till exactly.' (80)

'There are also those who have to be keenly aware of the meaning of measurements, even though they seldom measure anything for themselves. These include staff who are involved in ordering and costing, in calculating turnover and profit margins. Here again, measurement is seldom exact but needs to operate within specified degrees of accuracy. This concept of measurement as something which is sometimes exact but more often needs to lie within stated limits is one which is very different from that which is commonly encountered in the classroom; it is also one which requires a conceptual understanding which takes time to develop.' (81)

METRIC AND IMPERIAL UNITS OF MEASUREMENT

'We have received a number of submissions which draw attention to the fact that Imperial units are still used in many parts of industry. Although there are many companies which use only metric units, others still operate mainly with Imperial units and many companies use a mixture of both metric and Imperial. There are also certain companies which make use of units which are peculiar to a particular industry. There are a number of reasons for the continuing use of Imperial units. Among the most significant are the expense of re-equipping workshops with new machinery and instrumentation, the need to maintain a supply of spare parts which conform to standard Imperial sizes and the needs of customers overseas who still make use of Imperial measures. However, where Imperial units are still in use, only a limited range is normally encountered on any particular job, for example yards or feet and inches, pounds and ounces. The use of fractions of an inch and of the 'thou' (thousandth of an inch) is also common. Where conversion from Imperial to metric units, or vice versa, is necessary it is usual to make use of a conversion table.' (82)

IMPLICATIONS FOR THE CLASSROOM

'\n
'It is of fundamental importance—and, we believe, not as self-evident as some might suppose—to appreciate the fact that all the mathematics which is used at work is related directly to specific and often limited tasks which soon become familiar. The different aspects of mathematics which are used are related to each other by their context so that their application is immediately evident. Increasing
experience of carrying out calculations or measurements helps to develop skills of estimation and approximation and an awareness of whether or not a result is sensible.' (83)

'However great the effort which is made, illustrations of the practical applications of mathematics within employment which are given to a group of pupils, whose members will enter many different types of jobs, cannot provide the immediacy of the actual job itself. Nevertheless, it is important that the mathematical foundation which has been provided in the classroom should be such as to enable competence in particular applications to develop within a reasonably short time once the necessary employment situation is encountered.' (84)

'The preceding paragraphs give some indication of the kinds of mathematical skill and understanding which are needed. We believe that it is possible to summarise a very large part of the mathematical needs of employment as 'a feeling for measurement'. This implies very much more than an ability to calculate, to estimate and to use measuring instruments, although all of these are part of it. It implies an understanding of the nature and purposes of measurement, of the many different methods of measurement which are used and of the situations in which each is found; it also implies an ability to interpret measurements expressed in a variety of ways.' (85)

'If boys and girls are to leave school equipped in this way, they will need, in the mathematics classroom and elsewhere, to have taken part in a wide variety of appropriate mathematical activities and to have discussed these at length with their teachers and with each other.' (86)

LIAISON BETWEEN SCHOOLS, TAFE AND INDUSTRY

The Committee believes 'that much more three-way co-operation between the school, further education and industry sectors should take place in a variety of ways.' (115)

'It is clear that over the last few years a good deal of effort has been put into liaison activities of various kinds. However, if the improvements achieved by the work of local groups and by other local initiatives are to be maintained, continuing discussion is needed; resources, too, must be maintained and where at all possible improved. It is not the case that discussion followed by the publication of a report, syllabus or test paper will set things right once and for all. All parties must continue to learn from each other. To maintain a continuing discussion is perhaps more demanding than the initial effort required to produce a document of some kind. It also makes a continuing demand on the time of the teachers, industrialists and others who are involved. Local education authorities, careers advisors, teachers and employers will all need to take responsibility and initiative for the various aspects of liaison.' (116)
THE MATCH BETWEEN SCHOOL, EMPLOYMENT, AND TAFE

'Ideally a school leaver should experience continuity of learning in mathematics after moving from school to further education and employment. However, this is by no means a simple process.' (162)

The Committee drew attention to the fact that in the U.K. the mathematical demands of Further Education (i.e. TAFE) courses are likely to be considerably more than the demands of the job itself.

'One reason can be that in some cases it is necessary to go beyond immediate requirements in order to develop confidence and familiarity with essential topics. It is also the case that many courses are intended to provide not only the specific skills which are needed in the early years of employment but also a base for forty or more years of working life. Nevertheless, mathematical skills which are not used regularly can very easily atrophy, especially if they have proved difficult to comprehend, and so may not prove to be available when they are needed. It seems also to be the case that promotion can often lead to the use of less mathematics rather than more, because time is spent on supervisory and other duties.' (163)

'The diversity of school syllabuses can also lead to problems of mismatch between college courses and courses which have been followed at school.' (164)

'Although the range of mathematical ability can be particularly wide among those on craft courses, the fact that mathematics is not usually examined separately on CGLI courses makes it possible for students to avoid the more mathematical questions in examinations and still obtain good grades. This would seem to underline the fact that many craftsmen need to use only a very limited range of mathematical skills.' (165)

'The Bath and Nottingham studies found that attitudes towards mathematics among students at FE (i.e. TAFE) colleges were very often more favourable than had been the case when they were at school. In some instances this seemed to arise from the fact that the applications of mathematics were more immediately apparent. Even when this was not the case there were some who persevered because they felt that the mathematics they were having to learn was certain to be needed at some stage or it would not have been included in the college course.' (166)
CHAPTER 4

A RESPONSIVE PROCESS FOR FACILITATING CHANGE

4.1 INTRODUCTION

Educational researchers in the U.S.A., U.K. and Australia have increasingly been focusing on the processes involved in facilitating change. Knowing 'what' needs to be changed is not sufficient; we must also know 'how' to change.

In the U.S.A., the National Council of Teachers of Mathematics (NCTM) has recently published an Agenda for action: Recommendations for school mathematics of the 1980s. However, the NCTM recognised that mathematics educators would need help in implementing these recommendations. To meet this need, the NCTM commissioned a professional reference book on change, Changing school mathematics: A responsive process (Price and Gawronski [Eds], 1981).

Changing school mathematics is presented in three sections:

1. Changing Schools
2. Changing Mathematics Programs
3. Changing, and Being Changed by Others.

Two papers in section 1 are particularly relevant to the 'mathematics in trade courses' project in TAFE. The first is a paper by Kenneth H. Blanchard, Blanchard Training and Development, San Diego, California, and Patricia Zigarmi, National Staff Development Council, Oxford, Ohio on 'Models for change in schools'; and the second paper 'Supporting classroom change' by Ann Lieberman, Teachers College, Columbia University, New York, and Lynne Miller, South Bend Community Schools, South Bend, Indiana. The following section is based on these papers.

4.2 A BRIEF REVIEW OF RESEARCH ON CHANGE

Berman and McLaughlin (1975) report on a study of federal programs supporting educational change which was undertaken to determine what needed to be done for successful change. The study identified four broad factors influencing the successful implementation and continuation of change efforts in schools:

1. institutional motivation
2. implementation strategy
3. institutional leadership
4. teacher characteristics.
**Institutional motivation**

The study found that teachers must be committed to an idea for it to be successfully implemented. Teachers' commitment is influenced by the motivations of administrators, the planning strategies used in the project, and the scope of the project. For example, if teachers see that administrators are not supportive of a new idea then many are unwilling to invest their time and energies.

Secondly, collaborative planning brought about the consensus and support of all those involved in change efforts. The study found that teachers usually have little personal investment in project objectives or success when they have not participated in planning.

Finally, 'complex and ambitious projects were more likely to elicit enthusiasm of teachers than were routine and limited projects' (Berman and McLaughlin, 1975, p.vii). That is, the scope of the change proposed by the project influenced teacher motivation.

**Implementation strategy**

The study found that 'skill specific' training alone did not ensure long-term change in teachers or the continuation of the project.

The researchers found that three activities helped teachers adapt new ideas to their individual classrooms:

1. teachers received in-classroom assistance from credible resource people
2. teachers had an opportunity to participate in decision making
3. regular staff meetings were held to solve problems.

**Institutional leadership**

Berman and McLaughlin (1975) also found that the support of central office personnel and, even more, the support of the principal are essential if successful change is to take place. The active leadership of principals seemed to influence the quality of working relationships among teachers and promoted the continuation of the project's methods and materials.

**Characteristics of teachers**

The study found that the more experience a teacher has, the less likely the project was to achieve its goals and the less likely the project was to improve student performance.

The most powerful attribute contributing to successful implementation was the teachers' *sense of efficacy*—the belief that the teacher can help even the most difficult or unmotivated student.
4.3 THE PROCESS OF CHANGE

Kurt Lewin (1951) developed a general model for understanding the process of change.

The first phase is unfreezing, which is a thawing-out process in which the individual or group is motivated so that they see the need for change.

Next is the changing process which takes place after people are ready to change and incorporates the steps that are taken to make the innovation work in a particular setting.

The final phase is refreezing—the systematic reinforcement and support of changes that have occurred during the change process.

The problem with most change efforts in schools is that the unfreezing and refreezing processes never take place. A new program is merely thrust on the school before anybody is ready or willing to engage in it. (Blanchard and Zigarmi, 1981, p.41)

4.4 DIAGNOSTIC TOOLS

Determining how much unfreezing needs to be done

Force field analysis, developed by Kurt Lewin (1951) is very useful in determining how much unfreezing needs to take place in the school setting before the innovations can be effectively implemented.

Blanchard and Zigarmi (1981) suggest that before embarking on any attempt to implement an innovation, the people responsible for the change effort should determine what they have going for them (driving forces) and what they have going against them (restraining forces).

Examples of driving forces might be support from the central office and the principal, commitment of teachers, and a sense of efficacy on the part of many teachers.

Restraining forces are those acting to block the change; for example, lack of administrative support and a commitment to the old way of doing things.

In essence, when the forces that are pushing for change are counterbalanced by forces resisting change, there is no movement.

Assessing the attitudes of teachers

Hall, Wallace and Dossett (1973), at the Research and Development Centre for Teacher Education at the University of Texas, Austin, developed the 'Concerns Based Adoption Model' based on several assumptions about change:
change is a process not an event

educational change is a personal experience

although teachers' concerns vary in intensity and duration throughout a change effort, they usually diminish as the teachers become more familiar and skilful with the innovation.

A model of interventions has been evolving from the projects at the Research and Development Centre for Teacher Education. In the model, facilitators are advised to plan and carry out interventions designed to do the following:

1. develop supportive organisational arrangements
2. provide training
3. provide consultation and reinforcement
4. monitor and evaluate the change effort
5. communicate and disseminate information about the innovation.

Diagnosing antecedents and consequences of change

Blanchard and Zigarmi (1981) emphasise that, 'Even when teachers understand an innovation and have a positive attitude toward it, this does not guarantee that they will be able to change their behaviour in the classroom' (p.47).

They conclude that... 'the key to real long-term implementation and incorporation of an innovation is managing the consequences' (p.47).

If innovators do not positively reinforce individuals for attempting behavioral change in the early stages of an innovation and, instead, ignore them or pay them little attention, then the frequency of the new behavior will diminish ... The key to effective implementation of change is to "catch people doing something right" early in the intervention ... [The] frequency of the reinforcement schedule should start to diminish gradually over time until eventually users are comfortable with the change and are able to provide their own reinforcement. (Blanchard and Zigarmi, 1981, p.49)

4.5 SUPPORTING CLASSROOM CHANGE

Lieberman and Miller (1981) in their paper 'Supporting classroom change' emphasise that it is important that:  

32
Reformers should assume from the outset that it is teachers themselves who make change happen and make it endure ... it is important that teachers be actively engaged in the improvement process and that they see the connection between what they are trying to do and what effects those attempts have on students ... [however] Without an organizational commitment to, and engagement in, improvements, individual efforts by teachers in isolated classrooms do not hold much promise for sustained success. (Lieberman and Miller, 1981, pp.52-54)

Luckily, some of the more promising approaches now emerging assure us that change is indeed possible. Among these approaches are:

- staff development
- networking and dissemination.

4.6 STAFF DEVELOPMENT

Lieberman and Miller (1981) point out that staff development can be viewed as either a remedial strategy to improve the teaching practice of individuals or a viable approach to school change. Miller and Wolf (1978) have suggested a staff development approach based on research and theory. This approach can be represented by the following:

The staff development/school change model

```
individual concerns
  ↓  ↓
individual actions
  ↓  ↓
dialogue about actions
  ↓  ↓
collaborative action
  ↓  ↓
organizational change and support for change
```


Initially, the facilitator works with individual teachers on their own concerns through such activities as seminars, classes, workshops, consultations, and/or in-class assistance.

The teachers are encouraged to try new ideas in their own classroom. After they have had some success, they are encouraged to share their experiences with others.
Discussion groups are set up so that teachers can talk about issues of mutual concern and possibly plan joint projects. At this point the support and leadership of the principal is essential. Structure and support for these discussions can be provided through seminars, workshops and so on.

The school begins to change as these collaborative actions are tested. An environment supportive of individual change and incorporating organisational change is thus created. The above process is cyclical and staff development can begin at any point.

4.7 NETWORKING AND DISSEMINATION

Lieberman and Miller (1981) conclude that:

The large scale studies of school improvement in the last decade have revealed that it is necessary to provide continuing support for teachers and principals as they work in improving their schools. (Lieberman and Miller, 1981, p.56)

Networks can provide sources of support different from those found within a single school. Networks can be either formal or informal, but have five ingredients:

. A sense of being an alternative to established systems . . .
. A feeling of shared purpose
. A mixture of information sharing and psychological support
. A person functioning as a facilitator

The focus of the network helps to bind people together and build commitment. Because the concerns of the network are the concerns of the participants, people come to meetings already interested.

Networks depend also on good interpersonal relations. 'The very act of providing meetings and getting people together in an informal way on neutral ground can be a significant step toward engaging and supporting people and sharing resources'. (Lieberman and Miller, 1981, p.57)

Someone needs to be in charge of organising the network:

This leader's major tasks are to persuade people to interact with each other and to share their knowledge and experience. He or she needs to know when to be directive (provide information) and when to be permissive (provide opportunities for the group). Building the group feeling is more important than building the leader's expertise. (Lieberman and Miller, 1981, pp.57-58)
Emrick and Peterson (1978) have synthesised the findings of several studies where the major goals were to get new ideas into a school and have them understood, adopted, and used. Lieberman and Miller (1981) summarised their findings as follows:

- Interpersonal influences in involving people in an improvement strategy appear to be critical.
- The style of both the . . . [facilitator] . . . and the staff and their capacity to work together is more important than substance or expertise.
- Local involvement and commitment appear to be most critical in getting the process going.
- Training that is significant involves face-to-face continuous personal involvement. (p.58)

Lieberman and Miller (1981) add that . . . 'Structures can be created that do not need money or fancy buildings. They do need a focus, voluntary participation, informality, flexibility, and provisions for information and support' (p.58).

4.8 IMPLICATIONS FOR TAFE AUTHORITIES AND MATHEMATICS IN TRADE COURSES

Based on the above discussion, it is suggested that the possible advantages be explored of facilitators being employed within TAFE Authorities to initiate and co-ordinate staff development programs in teaching and learning mathematics in trade courses. In particular, facilitators could plan and carry out interventions designed to:

1. develop supportive organisational arrangements
2. provide training in approaches to teaching mathematics in trade courses
3. provide consultations and reinforcement
4. monitor and evaluate efforts to improve mathematics teaching in trade courses
5. communicate and disseminate information about this project.

The facilitators could work with individual trade teachers on their own concerns, encouraging them to try new ideas in their own classrooms.

Discussion groups could be set up so that trade teachers can talk about issues of mutual concern and possibly plan joint projects.

Continuing support for trade teachers could be provided. For example, networks of trade teachers across colleges could be set up.
5.1 INTRODUCTION

This section is intended for trade lecturers and remedial teachers. It should be emphasised that what is outlined in this section is not intended as a prescriptive guide nor meant to define exactly how to teach any mathematical concept. It is one of a number of possible approaches to teaching mathematics to trade students.

It is important to appreciate that all mathematics used in the workshop is related directly to specific and often limited tasks which soon become familiar. The different aspects of mathematics which are used are related to each other by the context so that their application is immediately evident.

Increasing experience of carrying out calculations or measurements helps to develop skills of estimation and approximation and an awareness of whether or not the result is sensible.

It is possible to summarise a very large part of the mathematical needs of trade students as a 'feeling for measurement'. This implies an ability not only to calculate, to estimate and to use measuring instruments, but also an understanding of the nature and purposes of measurement, and an ability to interpret measurements expressed in a variety of ways.

5.2 TEACHING FOR CONCEPTUAL UNDERSTANDING—GUIDELINES

The approach suggested here is based on teaching for conceptual understanding and on the other implications of the research findings listed in Chapters 2 and 3.

The suggested guidelines are intended only to be suggestions—to be utilised as relevant in a flexible and thoughtful manner, to meet the needs of the teacher, the learner, and the specific content to be learnt.

a. Use a 'real' problem based on a familiar situation

Choose a simple problem which emphasises the application of the concept you are trying to teach. Don't complicate the problem by introducing large numbers, numbers with decimals, or unfamiliar units. Use common well-known units, so that the presumed knowledge required of these units does not raise the conceptual difficulty of the problem. Initially use small whole numbers. Preferably concentrate on developing a feeling for the relationships involved, for example, larger or smaller than the original, or doubling one dimension and seeing what happens.
b. **Find out what the student already knows**

If the student already has a feeling for the relationships, based on previous experience, introduce the units of measurement.

If the student does not have a feeling for the relationship, you need to provide some practical work for him so that he can develop this 'intuitive' feeling.

c. **If the student does not have an intuitive feeling for the relationship involved, let the student gain that 'prior knowledge'**

Arrange an experiment in the workshop for the student, which will illustrate the relationships. Make connections between what the student knows, the experiment, and the problem. Give immediate feedback of correctness. Discuss any misunderstandings.

d. **Introduce other real problems**

Extend the student's understanding of the relationships and concepts by introducing other practical problems.

Find out what the student already knows, arrange practical experiments as before. Discuss the problem, encourage the student to discover the relationships for himself.

e. **Introduce units of measurement**

After the student has developed an understanding of the relationships based on 'unit-free' examples, introduce common units, and then less familiar ones.

Let the student get a feel for the units of measurement he will be using, by seeing them in context in the workshop; for example, piston areas are measured in square millimetres not square metres, pressure is measured in pascals not kilopascals.

If you measure, say pressure, with a gauge, let the student get a feel for different pressures using the gauge. Describe the different pressures in terms of the units used on the measuring device.

f. **Start to quantify the relationships**

Instead of describing a relationship in terms of 'larger than' or 'smaller than', introduce small whole numbers. Quantify the relationship by trying some simple experiments with known quantities. Write down the results of the experiment so that a pattern can be seen.
g. **Encourage the student to describe the relationship in his own words—to write down a rule**

If the student discovers the rule for himself, and convinces himself that it is true, it will be meaningful and relevant to him in that context.

Encourage the student to test whether he can generalise from his rule to other situations.

h. **Introduce the 'accepted terminology'**

Once the student has a feel for the relationships, and has developed a rule to describe the relationship, you can introduce the terminology used in the trade, making sure you maintain the connections between the terms and symbols you introduced and the meanings known to the student.

i. **Consolidate and practise fundamental skills and routines**

Once the student has a feeling for the concepts, the units of measurement involved, the relationships that exist, and the applications, you can then consolidate this by extending the range of problems and by practising fundamental skills and routines.

If you start by introducing a formula, and practise fundamental skills and routines, the student may become quicker at the skills but may not understand when to apply the formula. He may not be able to transfer these skills to practical situations. The above method of conceptual teaching has been developed to overcome this problem.

5.3 **AN EXAMPLE: HYDRAULIC BRAKE SYSTEMS**

a. **Use a 'real' problem based on a familiar situation**

On a certain make of vehicle the rear wheel cylinders on the station wagon differ in diameter from those of the car. This is because the station wagon has more weight on the rear wheels and therefore more braking force can be applied before wheel lock-up occurs. An owner of a car of the above make brings it in to you complaining that he has recently had new wheel cylinders fitted while on an interstate trip and now the rear wheels seem to lock-up too readily when he brakes heavily. You suspect the wrong diameter wheel cylinders may have been fitted and therefore check their diameter. Since the car would not require as much braking force as the station wagon, which wheel cylinders should have been fitted—the larger or the smaller diameter?
b. Find out what the student already knows

If the student already understands the relationship between 'piston area' and 'output force' then you can proceed by introducing the units of measurement and by quantifying the relationship.

If the student does not understand the relationship between 'piston area' and 'output force' then you need to teach him this concept before proceeding.

c. If he does not understand the concept, let the student experiment in the workshop with brake cylinders with different piston areas

Let's say you have two brake cylinders with different piston areas in the workshop.

Let the student experiment: encourage him to find out the relationship between piston area and output force by applying equal pressure to each piston and observing the output forces.

Discuss the problem

Ask the student to tell you about his experiments. Which cylinder had the larger output force? Which had the smaller output force?

Give immediate feedback of correctness—discuss any misunderstandings

Ask the student to describe the relationship between piston area and output force for the same pressure—get him to describe it in his own words. For example, the larger the piston area the larger the output force, for the same pressure.

Make connections between what the student knows and the original problem—relate the student's 'terminology' to the 'accepted terminology' used in the trade. Make sure he knows what the 'accepted terms' mean in the problem situation.

Can he answer the original question?

d. Introduce other 'real' problems

Introduce other 'real' problems so that the student can discover other relationships between piston area, pressure and output force—always keep one constant.

For example, experiment with cylinders with the same piston area. What happens to the output force when you increase the pressure? Ask the student to describe the relationship in his own words, then introduce accepted terminology.

e. Introduce units of measurement

Do you use a gauge to measure pressure? What units are on the pressure gauge? . . . Kilopascals
Let the student get a feel for different pressures in terms of the number of pascals?

The area of brake pistons is usually measured in square millimetres. Let the student get a feel for the size of different pistons in the workshops in terms of square millimetres.

Do you have a gauge to measure 'force'? What units are on the gauge? ... Newtons

f. **Start to quantify the relationship between pressure, piston area and output force**

Let the student use the gauges and experiment with different pistons and pressures—use small, whole numbers. What is the output force from a cylinder if you apply a pressure of 1000 kilopascals to a piston with an area of 200 square millimetres? ... 200 newtons.

Pick a cylinder with an area of say 400 square millimetres. What is the output force here for a pressure of 1000 kilopascals? ... 400 newtons.

If you double the pressure on the same piston, will the output force increase or decrease?

Apply a pressure of 2000 kilopascals to the 400 square millimetre piston. What is the force? ... 800 newtons.

If you apply a pressure of 3000 kilopascals to the 400 square millimetre piston the output force is 1200 newtons.

Can you see a pattern?

If you apply a pressure of 4000 kilopascals to the 400 square millimetre piston the output force is 1600 newtons.

g. **Encourage the student to work out a rule to describe this relationship**

<table>
<thead>
<tr>
<th>Number of newtons</th>
<th>Number of kilopascals</th>
<th>Number of square millimetres</th>
</tr>
</thead>
<tbody>
<tr>
<td>(in hundreds)</td>
<td>(in 1000s)</td>
<td>(in hundreds)</td>
</tr>
</tbody>
</table>

Is this rule true all the time?

Experiment with other cylinders and pressures.

If the student discovers the rule for himself and convinces himself that it is true, then it will be meaningful and relevant to him.

h. **Introduce the 'accepted terminology'**

Once he understands the concept, you can introduce the 'accepted terminology'. Make sure you maintain the connections between the symbols you introduce and the meanings already known to the student.
So 'pressure' is measured in kilopascals
'force' is measured in newtons
'area' is measured in square millimetres

We can rewrite our rule above as
Force = Pressure \times Area

i. **Consolidate and practice fundamental skills and routines**

Once the student understands how the concepts of pressure, force and area, relate to the problem, understands what the symbols mean, and what the formula or rule means, you can then consolidate this by practising fundamental skills and routines.

5.4 **AN EXAMPLE: AREA**

a. **Use a real problem based on a familiar situation**

You are asked to work out how many new tiles you need to replace the old tiles on the bottom of a spa bath.

b. **Find out what the student already knows**

If the student already knows how to tackle the problem you should encourage him to do so.

If he does not know how to start, you need to find out what he does know.

c. **Experiment with concrete materials**

Let's say you have a model of the spa bath, e.g. a cardboard carton with the base ruled into a $7 \times 10$ grid made up in the workshop.

**Discuss the problem with the student**

Perhaps he could count how many tiles are on the bottom of the spa bath at the moment.

He could get a piece of chalk or a marker pen, and could count the number of tiles starting from the top left hand corner. As he counted he could put the numbers on each tile.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>32</td>
<td>33</td>
<td>34</td>
<td>35</td>
<td>36</td>
<td>37</td>
<td>38</td>
<td>39</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>42</td>
<td>43</td>
<td>44</td>
<td>45</td>
<td>46</td>
<td>47</td>
<td>48</td>
<td>49</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>52</td>
<td>53</td>
<td>54</td>
<td>55</td>
<td>56</td>
<td>57</td>
<td>58</td>
<td>59</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>61</td>
<td>62</td>
<td>63</td>
<td>64</td>
<td>65</td>
<td>66</td>
<td>67</td>
<td>68</td>
<td>69</td>
<td>70</td>
<td></td>
</tr>
</tbody>
</table>
Discuss the problem. Can the student see any pattern in the numbers he is writing on the tiles?

- How many tiles are there in 1 row? 10
- How many tiles are there in 2 rows? 20
- How many tiles are there in 3 rows? 30
- How many tiles are there in 7 rows? 70

Can he see an easier way to work out the number of tiles without counting them all?

Ask the student to describe the relationship between the number of tiles in one row, the number of rows, and the number needed to cover the bottom of the spa bath.

Number of tiles needed to cover the bottom of the spa bath = Number of tiles in one row \times Number of rows

Make connections between what the student knows and the original problem

So how many tiles do you need to replace the old tiles?

Can he answer the original problem

If there had only been 3 rows, each with 10 tiles, how many tiles would he need?—generalise

d. Introduce other real problems

Someone is building a new spa bath, and they want you to work out how many tiles to buy to cover the bottom.

The bottom of the spa bath is 300 centimetres long and 200 centimetres wide.
The tiles they want to buy are all the same size—the tiles are 10 centimetres long and 10 centimetres wide.

Discuss the problem

Make connections between what the student already knows and the problem.

He knows that:

Number of tiles to cover the bottom of the pool = Number of tiles in one row \times Number of rows

Can he describe the relationship between the length of the pool, in centimetres, and the number of tiles in one row?

1 tile is 10 centimetres in length
2 tiles are 20 centimetres
3 tiles 30
4 tiles 40

Can he write down a relationship between the number of tiles in one row and the length of the spa bath?

Discuss any misunderstandings

Give immediate feedback of correctness

Length of the bath = Number of tiles in one row \times Length of each tile (in centimetres)

Can the student write down a relationship between the width of the bath, the number of rows of tiles, and the width of each tile?

Discuss any misunderstandings—make connections

e. Introduce units of measurement

The area of the bottom of the spa bath is usually measured in square metres.

If the bottom of the spa bath was 3 metres long and 2 metres wide what is the area of the bottom of the spa bath in square metres? Draw a diagram to represent the bottom of the bath.

\[
\begin{array}{c}
\text{2 metres} \\
\text{3 metres}
\end{array}
\]
Discuss the problem

Mark off lengths of 1 metre along the sides, and join up the sections.

Let the student get a feel for the size of 1 square metre, 2 square metres, and so on by constructing squares (and oblongs) of these sizes.

How many 'square metres' on the bottom of the spa bath? Give immediate feedback of correctness—discuss any misunderstandings.

Make connections between this and what the student already knows.

Encourage him to write down a rule in his own words, then introduce 'accepted terminology'.

\[
\text{Area} = \text{Length} \times \text{Width} \\
\text{(square metres)} \times \text{(metres)}
\]

Consolidate by practising on other problems.

Generalise to areas in other familiar contexts.

5.5 AN EXAMPLE: PLUMBING—AN INTRODUCTION TO PIPE SIZING

a. Use a real problem based on a familiar situation

You are called in to give a quote for replacing the pipes for a cold water system in a suburban house. The old pipes were installed 40 years ago and need replacing.

b. Find out what the student already knows

Does the student understand the concepts involved, for example; that as water passes through a pipe there is a loss of output force because of the friction of the water rubbing against the sides of the pipe?
c. **If the student does not understand the concept**

Let the student experiment with pipes of the same length but different diameters, connected to the same mains pressure or say a storage tank.

Let's say the student has two pipes connected to a tank. One pipe has a smaller diameter than the other.

Let the student experiment. Encourage him to find out the relationship between the diameter of the pipes and the output force for the same tank pressure.

**Discuss the problem**

Ask the student to tell you about his experiments. Which pipe had the greater output force? How could he get a feel for the output force?

Ask the student to describe the relationship between the diameter of the pipe and the output force at the end of the pipe, for pipes of the same length.

The output force at the end of the pipe with the larger diameter is greater than the output force at the end of the pipe with the smaller diameter when the pipes have the same length and are connected to the same mains pressure.

d. **Introduce other 'real' problems**

Can the student discover the relationships between length of pipe and output force, for the same diameter pipe.

- What happens to the output force when the pipe is longer?
- What happens to the output force when the pipe is shorter?
- Ask the student to describe the relationship in his own words and then introduce the accepted terminology.

e. **Introduce the concept of a 'balanced system' by having 2 pipes connected to the same mains outlet**

And so on.

5.6 **AN EXAMPLE: VOLUME**

You have to dig the footings for an extension to a house. You have to dig the trenches and arrange for the earth you excavate to be taken away.

You have a special trailer for your car. The trailer is 1m long and 1m wide. The trailer has sides which are 1m high. The trailer has a lid which fits on exactly on the top.
Let's draw a diagram to represent the trailer

![Diagram of a 1 metre long, 1 metre wide, and 1 metre high trailer]

You need to know how many trips you will need to make with the trailer to take away all the earth.

Let's say the first trench you have to dig is 5 metres long, 1 metre wide, and 1 metre deep.

Let's draw a diagram to represent the trench

![Diagram of a 5 metre long trench]

If you dig all the earth out of the trench how many trailer loads will it fill?

Start at one end of the trench. Let's say you marked off the length of the trench in 1 metre sections, so there are now 5 sections each 1 metre long, 1 metre wide and 1 metre deep.
You can number these section '1', '2', '3', '4', and '5'. If you excavate the first section, will all the earth fit into the trailer? Will there be any room left in the trailer?

The earth from the section 1m long, 1m wide and 1m deep fits exactly into the trailer 1m long, 1m wide and 1m deep.

So if you dig out 1 section it fits into 1 trailer load. If you dig out the next section it will also fit into 1 trailer load.

So 2 sections will be equivalent to 2 trailer loads.

3
4
5

The second trench you have to dig is bigger than the first one. It is 5 metres long, 2 metres wide and 1 metre deep

Let's draw a diagram to represent this trench.

If you dig all the earth out of the trench how many trailer loads will it fill?

Start at one end of the trench and mark off the length of the trench in 1 metre sections.

There are now 5 sections each 1 metre long, 2 metres wide and 1 metre deep.
Let's also mark off 1 metre sections on the width of the trench.

How big is each section now?
1m long by 1m wide by 1m deep.

How many sections are there? You can count them.

There are 10 sections each 1m by 1m by 1m.

So how many trailer loads are there? 10

The next trench you have to dig is deeper than the previous two

It is 5 metres long, 2 metres wide and 3 metres deep

Let's draw a diagram to represent this trench and divide it into sections 1m x 1m x 1m.
How many sections will there be?

You can't see them all in your diagram so you may miss some in counting.

Let's see if we can work out a rule to count them all?

On the first layer you can see them all, there are 5 sections in the length, and it is 2 sections wide, so there are 10 sections on the top.

The second layer should have the same number of sections as the first, so it has 10 sections as well.

So the total number of sections in 2 layers is $10 + 10 = 20$.

The third layer also has 10 sections, so the total number of sections in 3 layers is $10 + 10 + 10 = 30$.

Can you work out a rule to make this easier?

The total number of sections in 3 layers is $3 \times$ number of sections in 1 layer.

Can you work out a rule to calculate the number of sections in 1 layer?

The trench was 5 sections long and 2 sections wide, so the number of sections in 1 layer = number of sections long $\times$ number of sections wide.

Can you put both these rules together?

The total number of sections in 3 layers = $3 \times$ number of sections long $\times$ number of sections wide.

If the trench had been 8 sections deep the total number of sections = $8 \times$ number of sections long $\times$ number of sections wide.

In other words you have worked out a rule:

Number of sections in a trench = number of sections deep $\times$ number of sections long $\times$ number of sections wide.

If each section is 1 metre $\times$ 1 metre $\times$ 1 metre deep long wide then

number of sections = length $\times$ width $\times$ depth
(in cubic metres) (in metres) (in metres) (in metres).

50
Let's move on to a more difficult problem

After you have dug the trench, you have to order the concrete to put in the trench.

The concrete doesn't come in trailer loads, but in a big concrete truck. You have to order the correct quantity of concrete.

You order concrete in 'cubic metres'.

How much is one 'cubic metre' of concrete?

It is the amount of concrete that would fill a trench that was 1 metre long, 1 metre wide and 1 metre deep. It is the amount of concrete that will fill the first section of your trench which was 1 metre long, 1 metre wide, and 1 metre deep.

So how many cubic metres of concrete do you need to order?

You have already marked off your trench in sections—there were 5 sections each 1 metre long, 1 metre wide and 1 metre deep, so you need to order 5 cubic metres.
CHAPTER 6

SUMMARY

The present paper has attempted to highlight some issues related to mathematics in trade courses requiring further discussion, and to provide a basis for informed recommendations.

The following four themes were inferred from a variety of sources including the proceedings of a discussion group:

1. the need to be aware of recent research into teaching and learning mathematics
2. the need to refine the specific mathematical requirements of various trade programs
3. the need to address questions concerning content and teaching methods before using screening tests and setting up remedial programs
4. the need for continuing liaison among TAFE, State Education Departments, mathematics educators, and industry.

The analysis of issues concerned with mathematics in trade courses, and the reviews of research, suggested the need for reviews of the mathematical requirements and teaching methods in trade programs. In particular, the research into teaching and learning mathematics suggests that teaching methods based on conceptual understanding and diagnostic teaching are more effective than methods based on accumulating isolated skills or learning by rote. The research stresses the importance of context in learning and the difficulties associated with transferring skills to new problems.

In addition, the Cockcroft Committee found that 'all the mathematics which is used at work is related directly to specific and often limited tasks which soon become familiar' (Cockcroft, 1982, para. 83, p.24). TAFE Authorities need to consider the Cockcroft findings when they investigate the mathematical requirements of the various trade programs.

An outline of a responsive process for facilitating change has been presented. In this process, facilitators would be employed to initiate and co-ordinate developmental programs on teaching and learning mathematics in trade courses. In particular, the facilitators could plan and carry out interventions designed to:

- develop supportive organisational arrangements
- provide training in approaches to teaching mathematics in trade courses
- provide consultation about teaching approaches, mathematics education, and so on
- monitor and evaluate the efforts to improve mathematics teaching in trade courses
- communicate and disseminate information about innovations, for example, produce videos and other resources for use in staff development programs.
REFERENCES


Emrick, J., & Peterson, S. A synthesis of findings across five recent studies of educational dissemination and change. San Francisco: Far West Laboratory, 1978.


